Proceedings of Global Power and Propulsion Society
ISSN-Nr: 2504-4400
GPPS Hong Kong 2023
October 17-19, 2023
www.gpps.global

GPPS-TC-2023-0138

ON THE INFLUENCE OF INSTALLATION ON THE FORCED RESPONSE OF RADIAL TURBINE WHEELS

Alex Nakos
Brandenburg University of Technology
nakos@b-tu.de
Cottbus, Brandenburg, Germany

Bernd Beirow
Brandenburg University of Technology
beirow@b-tu.de
Cottbus, Brandenburg, Germany

ABSTRACT

Radial turbine wheels are commonly designed as integrally bladed rotors featuring extremely low structural damping in comparison to separate designs of blades and disk. Consequently, they are more prone to vibration. Moreover, random blade mistuning due to unavoidable inaccuracies in manufacture or material inhomogeneities can severely increase the maximum forced blade vibration amplitude compared to the tuned counterpart. Unfortunately, this response magnification may worsen in case of small damping. Since modes exhibiting blade dominated vibration are usually considered vulnerable in this regard, the influence of disk and shaft and its mounting conditions seems to be negligible. In this paper, reduced order models are employed in order to simulate the forced response of a radial turbine wheel. Experimental modal analyses have been carried out to provide realistic damping ratios considering both the single turbine wheel hardware as well as the full rotor mounted in a turbocharger test rig. Test runs are conducted and non-intrusive blade-tip-timing technology provides measurement data to validate the simulation models. Contrary to the original presumption, it is shown that additional structural damping contributed by assembling can significantly influence the forced response even though the focus is on blade dominated vibration.

INTRODUCTION

The requirement for more efficient, lightweight, and economical turbomachinery has led to radial turbine wheels that are designed as blade integrated disks (blisks). In comparison to rotors with separate designs of blades and disk, blisks are usually featuring a comparatively low structural damping of blade vibration. Thus, the system damping during machine operation is often reduced to the aerodynamic damping. The structural damping is even assumed to be negligibly small and consequently is either not considered at all or set very small in numerical simulations. In many instances the focus is on blade dominated modes considering the respective nodal diameters (ND) and assuming the structural damping to be almost equal since no significant participation of disk or shaft is expected.

The aim of this contribution is to investigate the influences of mounting conditions on the structural damping of a radial turbine wheel. Therefore, two wheels identical in design and taken from the same series have been considered in the framework of both numerical and experimental modal analyses as well as test runs on a turbocharger test rig. Wheel 2 remained in as-delivered condition and thus unmachined whereas Wheel 1 features a geometric adaption. Figure 1a exemplarily shows one of six machined blades of Wheel 1 in order to implement an intentional mistuning (IM) pattern of an AABB sequence. The total number of blades counts 12 so that half of the blades are machined according to Figure 1b. The IM-pattern was primarily intended to mitigate the forced response of blade mode (BM) 1 (first bending mode) due to an EO 10 excitation (Nakos et al., 2021; Nakos et al., 2022). Its positive impact on the BM 1 vibration of Wheel 1 could be proven by using non-intrusive blade-tip-timing (BTT) technology during test runs on a turbocharger test rig (Nakos et al., 2023). From the numerical point of view, reduced order models (ROM), namely Subset of Nominal System Modes (SNM) models (Yang and Griffin, 2001) were employed in order to efficiently simulate and predict the mitigation or magnification of the forced response. The respective input parameters such as mistuning, aerodynamic influence coefficients (AIC), speed, and operating temperatures have been measured or calculated for the two considered operating points. Indeed, the modelling allows to predict the response mitigation of BM 1 correctly, whereas deviations became apparent for a higher mode (BM 5) even though analogous simulation models have been used. The structural damping has been assumed to be negligible in both cases so that a comparatively low damping ratio was set uniformly for all modes. However, experimental analyses at resting wheels could show that the installation of wheels in the turbocharger increase the structural modal damping ratios of some modes significantly. Consequently, the simulation models were updated, and its results were compared to those received by evaluating the test run measurement data.

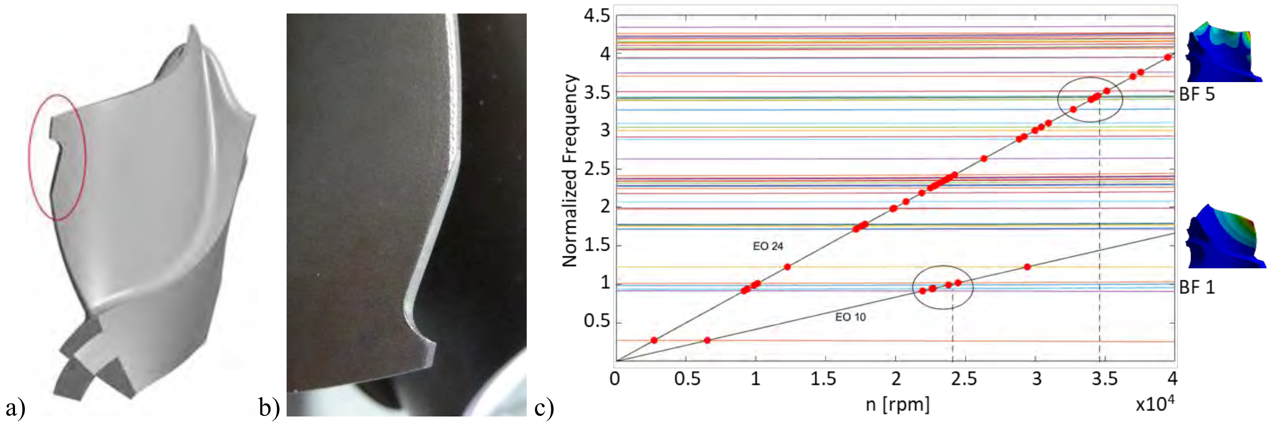


Figure 1: a) Modified Sector Volume Model, b) Machined Wheel 1 Hardware with IM-Modification and c) Campbell-Diagram

PRELIMINARY ANALYSES

In order to find a suitable IM-pattern that is able to drastically mitigate the forced response of the considered turbine wheel ROMs on basis of the SNM-theory originally introduced by (Yang and Griffin, 2001) were built up. The equation of motion in modal coordinates $\mathbf{q}(j\Omega)$ reads as follows.

$$[-\Omega^2\mathbf{M} + j\Omega\mathbf{D} + \mathbf{K} + \Delta\mathbf{K} + \mathbf{Z}]\mathbf{q}(j\Omega) = \mathbf{f}^E \tag{1}$$

Herein, $\mathbf{M}$ denotes the modal mass matrix, which corresponds to the unit matrix if mass-normalized mode shapes $\boldsymbol{\phi}$ have been used for the modal transformation. $\mathbf{D}$ represents the modal structural damping matrix, $\mathbf{K}$ the modal stiffness matrix of the tuned reference with centrifugal and temperature effects incorporated, and $\Delta\mathbf{K}$ accounts for mistuning in terms of blade-to-blade stiffness variations. The impedance matrix $\mathbf{Z}$ represents the aeroelastic coupling and results from transforming the cyclic influence coefficients matrix from blade individual coordinates into coordinates of the subset (Giersch et al., 2013). $\mathbf{f}^E$ stands for the external forcing.

Due to their ability to calculate the forced response within seconds and the low number of only 12 blades, all possibilities of arranging two different blade designs were worked out. The pattern AABB turned out to be the most effective in terms of mitigating the first bending mode (BM 1) and promised a reduction down to 54.4 % in comparison to the tuned counterpart and Wheel 2 (Nakos et al., 2021). This could be proved to be true as a mitigation of Wheel 1 down to 57.0 % was measured during test runs on the turbocharger test rig (Nakos et al., 2023). Figure 1c shows the Campbell-diagram for the considered turbine wheel on basis of numerical modal analyses. The pattern AABB did not seem to have such a positive impact on higher BMs. With a view to BM 5 which turned out to be critical during machine operation in the past (Figure 1c), only a slight mitigation of Wheel 1 down to 90.0 % was predicted. The test runs, on the other hand, even showed a reversed magnification of 6.4 % in relation to Wheel 2. This resulted in the question why the prediction for BM 1 forced response was very accurate and the one for BM 5 was not (Figure 1c). All investigations have been carried out analogously for both operating points in order to generate the respective input parameters for the SNM-models.

Aeroelastic Interaction

The SNM model simulates aeroelastic interaction using the AICs derived beforehand. In detail, unsteady CFD simulations have been carried out in order to calculate both aerodynamic damping and AICs depending on the travelling wave mode (TWM) for the two considered operating points. Figure 2a shows the results for blade mode family (BF) 1 vibration due to an EO 10 excitation and Figure 2b those for BF 5 vibration due to an EO 24 excitation. Please refer to (Nakos et al., 2021) for a detailed description of the simulation and calculation.

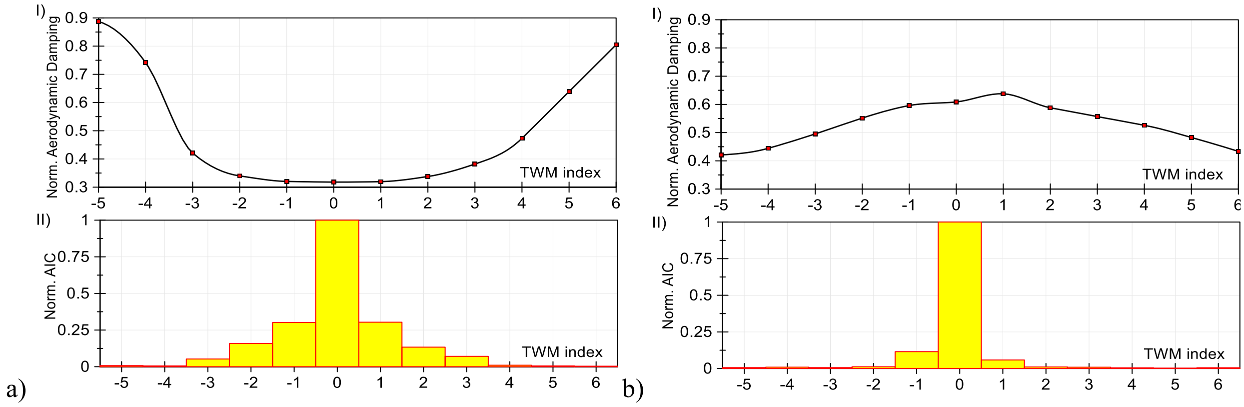


Figure 2: Aerodynamic Damping (I) and AIC (II) for a) BF 1 and b) BF 5 Calculated by ITSM Institute Stuttgart (Nakos et al., 2021)

If one compares the two damping curves in Figure 2 it can be seen that the aerodynamic damping strongly depends on the TWM. In case of EO 10/BM 1 one would excite a forward TWM 2 whereas in case of EO 24/BM 5 a TWM 0 is excited.

Mistuning ID and Experimental Modal Analyses

In order to be able to consider the actual mistuning of the wheel hardware in the framework of the forced response analyses, experimental modal analyses have been carried out. Therefore, frequency based experimental mistuning identifications were conducted for both wheels ("blade-by-blade measurements", Kühhorn and Beirow, 2010), considering BM 1 and 5. The wheel hardware has been excited by using a miniature modal hammer and the response has been measured using a laser Doppler vibrometer (LDV). On the one hand, the measurements have been carried out under laboratory conditions in order to represent a "free-free" support ("Laboratory (free)"). On the other hand, the measurements have been repeated analogously after installation on the turbocharger test rig ("Test Rig (mounted)"). Details of the wheel supports are visualized in Figure 4.

Figure 3 shows the mistuning identifications for BM 1 and 5 of both wheels. It can be seen that neither the mounting conditions nor the strain gauge (s/g) application have a significant impact on the blade mistuning. Please note that unfortunately there is no measurement data available representing BM 5 of Wheel 2 before s/g application (Figure 3b ii). The measured mistuning distributions at standstill have been adapted to operating conditions in order to be considered in the framework of the forced response simulations (Feiner, 2003 and Nipkau, 2010). The original IM-magnitude thus changed imperceptibly from $\Delta f\ = 1.0\ \%$ to $\Delta f\ = 1.086\ \%$.

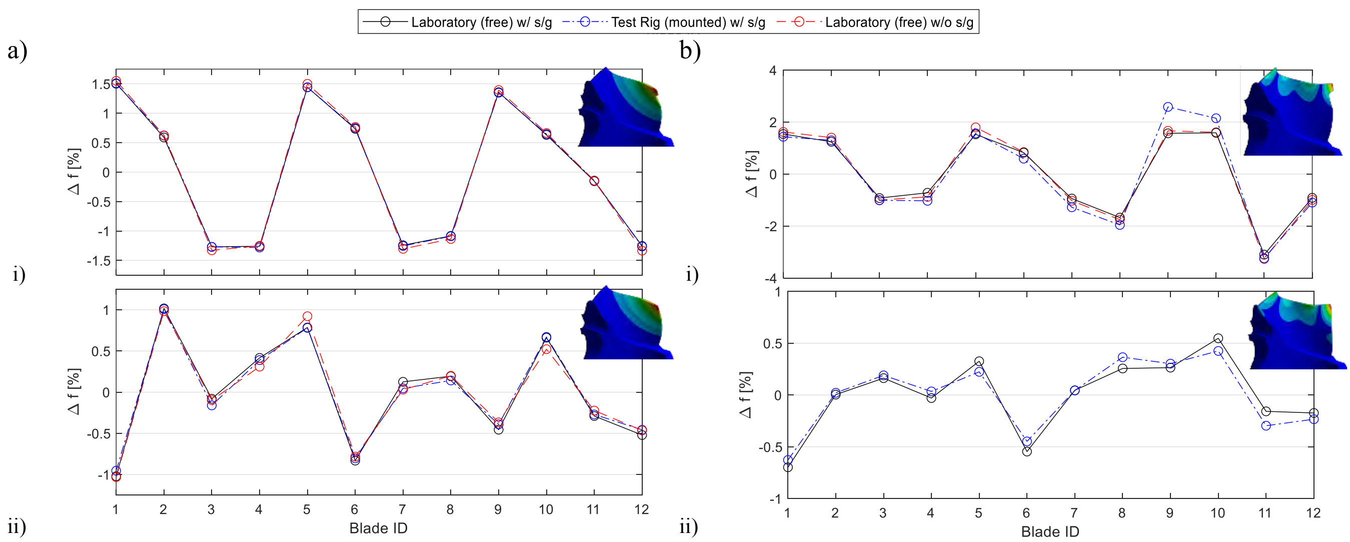


Figure 3: Mistuning Distribution for Wheel 1 (I) and Wheel 2 (II) Representing a) BM 1 and b) BM 5

In addition to the measurements mentioned above, conventional experimental modal analyses have been carried out in order to determine the modal parameters such as modal damping. The wheel hardware has been excited by using a miniature modal hammer and the response has been measured using a LDV. Similarly, to the presented blade-by-blade measurements, the experimental modal analyses have been carried out for both wheels under laboratory conditions ("free-free" support) as well as after installation on the turbocharger test rig.

Figure 4 shows the two measurement setups for both wheels. In the laboratory a wheel's shaft shoulder was supported on rubber pads in order to minimize the support area and to reduce energy dissipation due to friction (Figure 4a). This method of supporting the wheel, so to say, approximately enables what is known as a free-free support. Thereby, the shaft which is welded on the turbine wheel had a cantilever bearing and no contact to the surrounding at all. The modal hammer excited the structure by hitting on the pressure side's blade tip. The response was measured on the suction side's blade tip for every single blade separately. Thereby, the blade and location of excitation remained unchanged and 12 frequency response functions (FRF) were received. The FRFs have been evaluated using a MDOF-fitting algorithm in order to determine the modal parameters such as frequency, damping, and modeshape (Maia, 1988). Figure 5a exemplarily shows the FRFs of BF 1 for all blades of Wheel 2 in laboratory condition whereas Figure 5b represents the FRFs of the same wheel mounted in the turbocharger. Please note that the original modeshape and its associated ND usually gets lost when considering the mistuned wheel hardware and thus, various NDs take part. The modified nodal diameter (MND) denotes the original and still dominating ND superimposed with parts of other NDs of the same BF. In particular, the modes MND 0 and 1 are featuring a detectable frequency shift due to the mounting condition whereby the modes with MND 2 - 6 experience nearly no change in this regard.

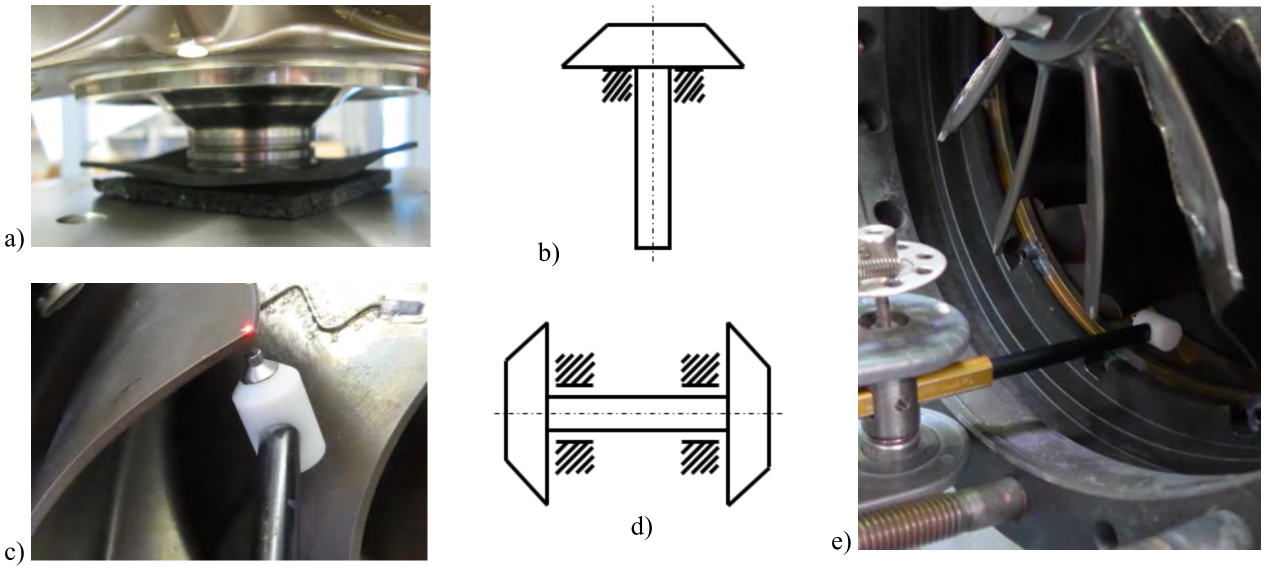


Figure 4: a), b) Support of Wheel in Laboratory, c) Excitation/Response and d), e) Mounted Wheel

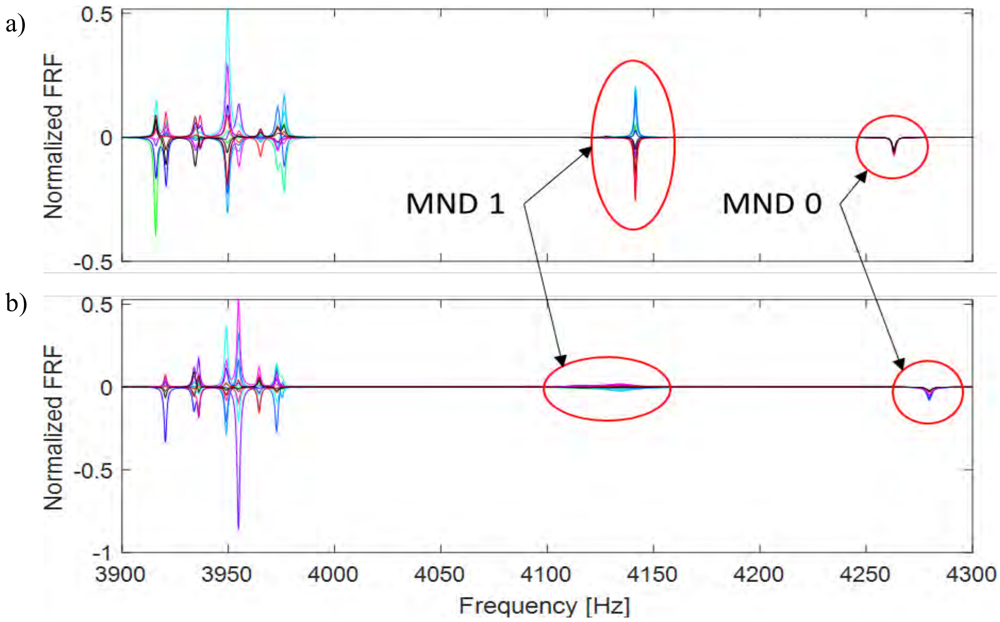


Figure 5: FRF of BF 1 of Wheel 2 (all Blades): a) In Laboratory, b) Mounted on Turbocharger

Contrary, the overall damping level is generally increased as a result of the mounting, and again the modes MND 0 and 1 are exhibiting a comparatively strong change. After evaluating all FRFs it was found that the modal damping for BF 1 increased by 78 % on average. The effect on the forced response was hardly recognizable due to the comparatively high aerodynamic damping shown in Figure 2a.

The analyses and evaluations have been repeated for BF 5 in order to investigate the effects of the mounting conditions. Figure 6a exemplarily shows the FRFs of BF 5 for all blades of Wheel 2 in laboratory condition and Figure 6b comparatively in mounting condition. In comparison to BF 1 (Figure 5) the average damping increase was not found as strong as the one for BF 1 but still takes a value of 37 %. However, the modes characterized by MND 0 and 1 are again featuring a noticeable frequency shift and strong increase in modal damping. The effect is even stronger in comparison to BF 1 as both modes almost completely disappeared after the mounting. The signal-to-noise ratio became such small so that a proper evaluation with the mentioned MDOF-fitting algorithms was no longer possible. Therefore, the modal damping ratios for both modes were estimated.

It is remarkable that in both cases, BF 1 and 5, the modes with MND 0 and 1 are featuring higher modal damping ratios in comparison to all remaining modes of the BF considered. They are also characterized by a strong frequency shift, whereby the remaining modes nearly hold their original frequency location. Hence, the mounting condition has a significant impact on all modes as it noticeably increases the modal damping in general and in particular for MND 0. In order to be able to simulate the forced response in a more realistic manner the actual structural damping has been considered in the SNM-models as well. The next section describes the SNM-models representing an EO 24 excitation.

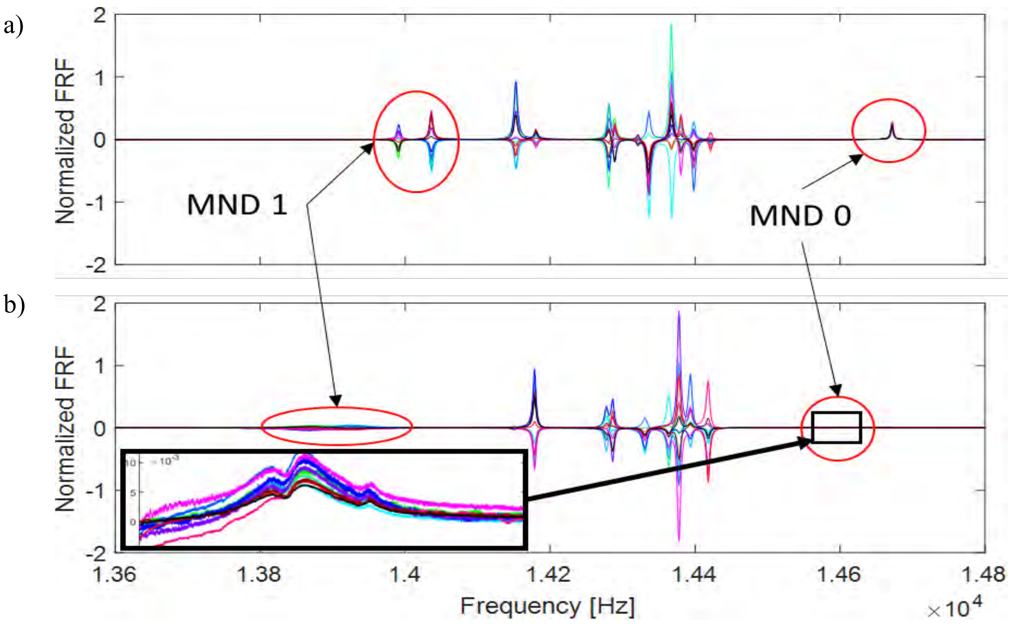
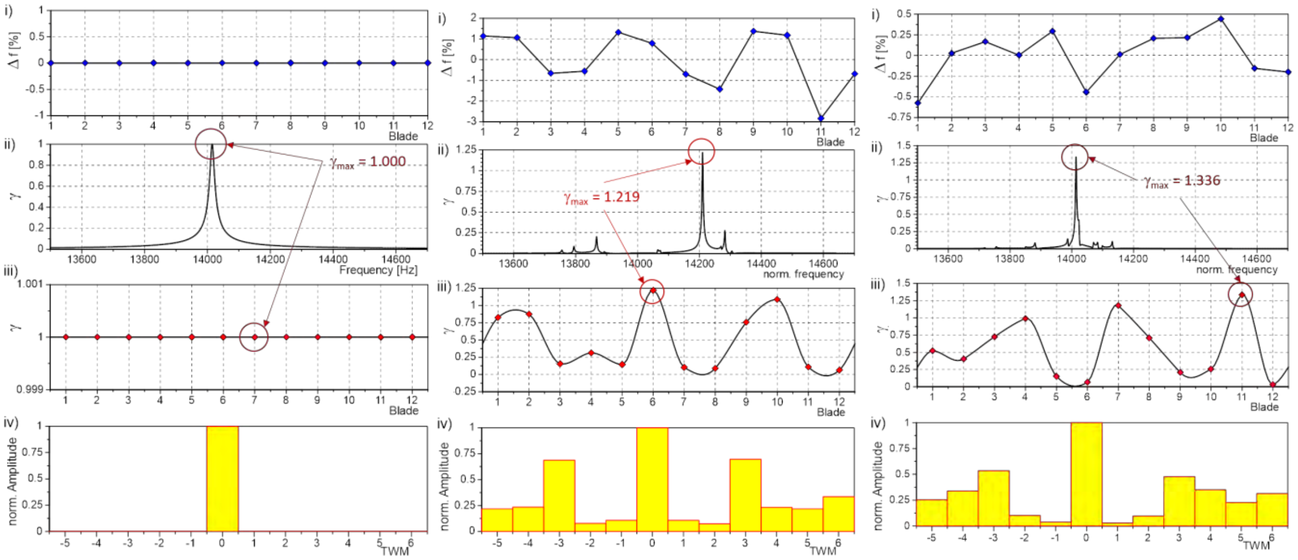


Figure 6: FRF of BF 5 of Wheel 2 (all Blades): a) In Laboratory, b) Mounted on Turbocharger

Forced Response Analyses

The forced response analyses have been carried out with the aid of ROMs, namely the SNM. The mitigation of BM 1 due to an EO 10 excitation thus successfully predicted could be proved during several test runs on the turbocharger test rig (Nakos et al., 2023). In contrast, the predicted mitigation of the maximum forced response of BM 5 could not be confirmed experimentally in the view of the fact that Wheel 1 showed a magnification of 6.4 % in relation to Wheel 2 instead. If one considers a negligibly low aerodynamic damping for all ND/TWM the following results for the two turbine wheels are to be expected (Figure 7). Hereby, Wheel 2 does not show the measured magnification but a mitigation of 8.8 %.

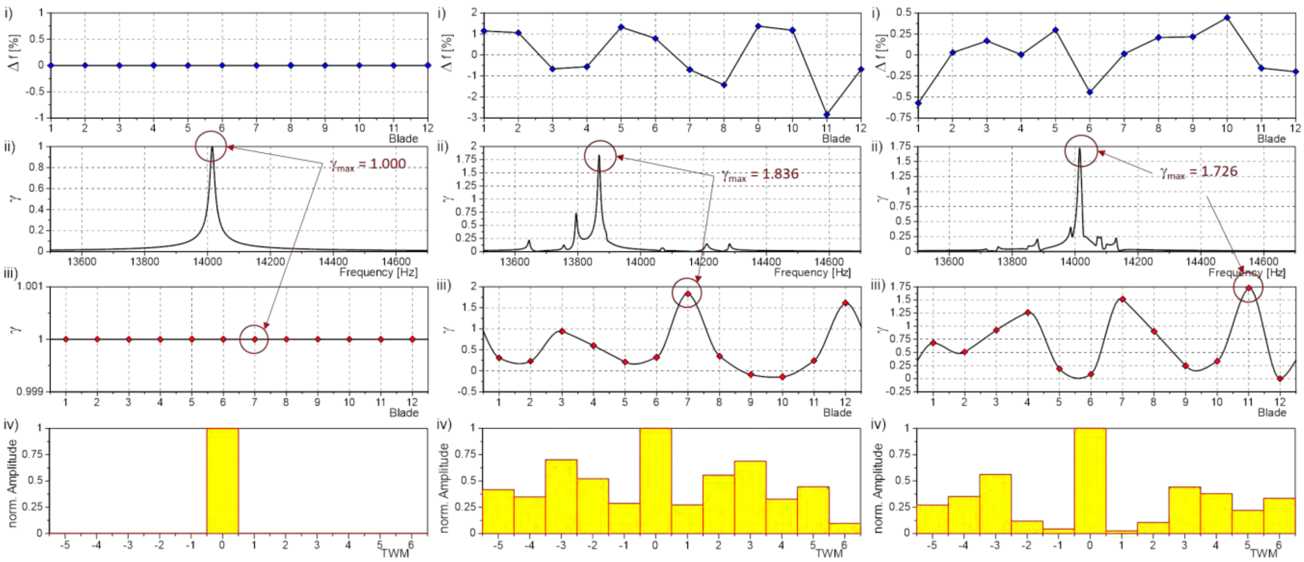
If one considered a theoretically tuned turbine wheel with identical blades and thus no mistuning at all, the operating deflection shape (ODS) would formed out as a pure “umbrella”-mode (MND 0) (γ_{max} = 1.000) due to the EO 24 excitation (Figure 7a iii). The TWM expansion delivers a clear and exclusive participation of the respective mode.



a) b) c)

Figure 7: i) Mistuning Pattern, ii) FRF of Critical Blade, iii) ODS, iv) TWM Expansion for: a) Tuned Reference, b) Wheel I, c) Wheel II

Since the modal damping of MND 0 was not properly evaluable (Figure 6b) a fourfold increase in relation to the MND 0 mode under free support conditions has been assumed. This was considered a realistic increase after analysing the FRFs. The other input parameters such as mistuning ID and AICs remained unchanged and were determined as described in the preceding section. Figure 8 shows the results of the adapted forced response analyses employing SNM-models.



a) b) c)

Figure 8: i) Mistuning Pattern, ii) FRF of Critical Blade, iii) ODS, iv) TWM Expansion for: a) Tuned Reference, b) Wheel I, c) Wheel II

The intentionally mistuned Wheel 1 featuring the pattern AABB shows a significant magnification of γ_{max} = 1.836 compared to the tuned counterpart and the original ODS gets lost (Figure 8b iii). There are further TWMs taking part at the response but nevertheless, TWM 0 forms the largest part (Figure 8b iv and c iv). Wheel 2, on the other hand, shows a slightly lower magnification of γ_{max} = 1.726 compared to the tuned counterpart (Figure 8c iii). Analogously to Wheel 1, further TWMs take part at the response whereby TWM 0 again forms the largest part. This corresponds to a magnification of 6.4 % of Wheel 1 forced response related to the one of Wheel 2.

Finally, this confirms the demand for a more precise definition of the modal damping for TWM 0 (ND 0). This targeted mode due to an EO 24 excitation stays dominant even if mistuning plays a role (Figure 8b iv and c iv).

EXPERIMENTAL SETUP AND TEST RUNS

In order to validate the forced response predictions employing the SNM test runs on a turbocharger test rig have been carried out. Therefore, the turbocharger has been integrated into the test stand at IKDG institute, RWTH Aachen University. Numerous test runs have been run crossing several resonances and operating points in order to validate the numerical analyses (Sasakaros et al., 2023; Schafferus et al., 2023).

Test Rig

The test rig consists of the turbocharger integrated into the plant at the IKDG institute. The turbine is run with exhaust gas and drives the compressor wheel conventionally as in regular turbocharger operation. Figure 9 shows the plant including the turbocharger under investigation.

Figure 9: Turbocharger Test Rig at IKDG Institute, RWTH Aachen University

The turbine inlet pressure can reach a maximum of $9.0\ bar$ and the maximum inlet temperature is $550\ °C$. The test runs have been carried out featuring an inlet temperature of $500\ °C$. Non-intrusive BTT-technology monitors both compressor and turbine wheel and records forced response data in the form of timestamps (TOA). The test rig has 16 measuring probes in total whereby 8 probes each monitor the compressor and the turbine wheel. Additionally, the compressor and both turbine wheels are applied with s/g to generate measurement values to be compared with BTT measurement data. However, the evaluation of s/g measurement data is not part of this paper. Details addressing this topic are given in (Schafferus et al., 2023) and (Sasakaros et al., 2023). The integral force excitation has been realized by using adapted inlet guide vanes (IGV) that are able to solely excite the targeted EO at the respective resonance crossing.

Test Runs and Measurement Results

In order to excite BF 5, the standard IGV has been installed and the turbocharger was run through a speed range that covers the resonance speed, namely $33{,}400\ rpm$ (Figure 1c). Three acceleration and three deceleration manoeuvres each identical to each other were run in sequence and BTT-measurement data, namely times of arrival (TOA) was recorded.

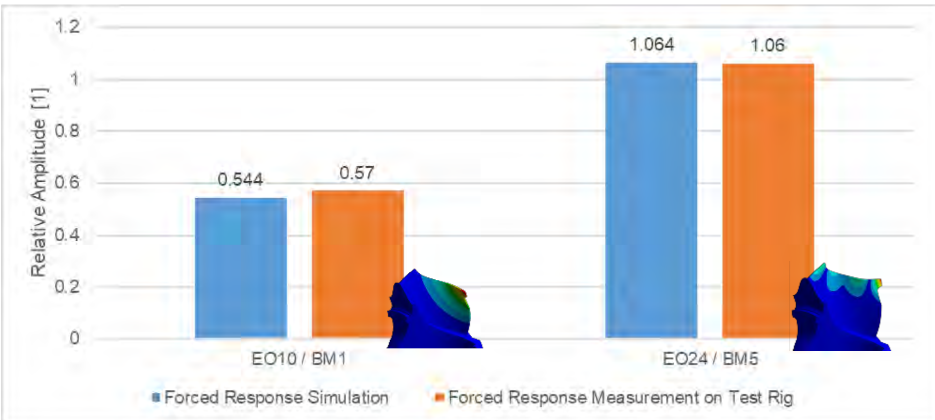


Figure 10: Simulated/Measured Maximum Vibration Amplitude of Wheel 1 in Relation to Simulated/Measured Maximum Vibration Amplitude of Wheel 2

Since the actual magnitude of the integral force excitation is not known and the SNM-models are simulated with unit loads, the measured blade deflections cannot be referred to the ones calculated. Thus, the measured deflections had to be related to one another. Figure 10 shows the measured/calculated maximum blade deflections of Wheel 1 referred to those of Wheel 2 for BM 1 and 5. The data represents the respective average of all six runs compared to the simulation results. The right bar (EO 24/BM 5) of Figure 10 confirms the predicted 6.4 % magnification of Wheel 1 blade deflection related to that of Wheel 2 based on the experimental results. Consequently, the SNM-models are able to accurately predict the forced response of BM 5 due to an EO 24 excitation if the actual structural modal damping is considered.

RESULTS AND DISCUSSION

The comparison between the preliminary numerical analyses regarding the turbine wheels' blade vibration and the test runs on the turbocharger test rig has shown a good agreement. The SNM-models representing EO 10/BM 1 forced response were featuring an estimated but comparatively low and thus negligible structural damping. The values were uniformly defined for all TWMs unlike the aeroelastic damping (Figure 2). However, since the aeroelastic damping is significantly higher than the structural damping the influence of the mounting condition was not detectable even though it has an impact on the structural damping. The evaluation of the test runs almost showed a coincidence between prediction and measurement.

In contrast, the SNM-models representing the EO 24/BM 5 forced response initially showed no satisfying agreement by predicting the magnification of the forced response incorrectly and wrong way round. Since the preparation and generation of input parameters have been done analogously, the assumption arose that the damping behaviour was not realistically defined. Therefore, numerous experimental modal analyses have been carried out in order to evaluate the influence of the mounting condition after the installation on the turbocharger. It was found that the structural damping significantly increased for MND 0 and 1. Actually, TWM 0 is the solely responding mode in the tuned case in consequence of an EO24 excitation and remains as main part of the response when mistuning is present. Moreover, the low overall aeroelastic damping level (Figure 2b) lead to an increased importance of setting the correct values for the structural damping. By solely assuming TWM 0 to be fourfold higher than the remaining modes of BF 5 the actual magnification of Wheel 1 related to Wheel 2 could be satisfactorily predicted.

CONCLUSIONS

This contribution investigates the impact of mounting conditions on the blade vibration behaviour of integrally manufactured radial turbine wheels (blisks). The focus is usually on blade dominated modes which feature a very low or no shaft vibration at all. The assumption is obvious, that the structural damping within a blade mode family (BF) is in a similar range for all nodal diameters (ND). In order to investigate the influence of an intentional mistuning (IM) pattern on a higher mode forced response analyses employing reduced order models (ROM) were carried out. Often, the structural damping plays a minor role when simulating blade vibration of blisks and was set negligibly small. In contrast to the first bending mode, the calculated aerodynamic damping of the investigated higher mode turned out significantly lower. This lead to the assumption that a precise knowledge of the structural damping becomes necessary. Initially, there was no reason to the assumption that the mounting conditions significantly influence the blade vibration behaviour. Contrary, the experimental modal analyses carried out have shown that the average damping level can very well be affected due to the impact of installation on the turbocharger and the modes with ND 0 and 1 show significantly increased structural damping ratios. The measured and evaluated damping ratios were considered in the framework of adapted forced response analyses. This allowed to calculate the exact blade magnification which was measured during test runs on the turbocharger test rig. An initial explanation can be given if one has a look at the respective modeshapes. A mode with nodal diameter (ND) 1 is featuring a very low inter blade phase angle (IBPA) and a mode with ND 0 is even featuring a IBPA of 0°. The blades vibrate in phase and thus the disk and shaft may follow with a torsional vibration. However, this behaviour is influenced by the changed shaft support when the rotor is mounted on the turbocharger. Further, the compressor wheel on the shaft end affects the shaft and turbine movement with its mass and moment of inertia which results in the observed frequency shifts. Future work will aim at simulating these effects employing FE-models by considering the full rotor system in order to be able to describe the exact damping effects. Furthermore, the modelling of the shaft support should be included as well.

NOMENCLATURE

Abbreviations:

AIC	Aerodynamic Influence Coefficients
BF	Blade Mode Family
BM	Blade Mode

BTT	Blade Tip Timing
EMA	Experimental Modal Analysis
EO	Engine Order
FRF	Frequency Response Function
IBPA	Inter Blade Phase Angle
IGV	Inlet Guide Vane
IM	Intentional Mistuning
LDV	Laser Doppler Vibrometer
MND	Modified Nodal Diameter
ND	Nodal Diameter
NMA	Numerical Modal Analysis
ODS	Operational Deflection Shape
ROM	Reduced Order Model
SNM	Subset of Nominal System Modes
TWM	Travelling Wave Mode
γ	Maximum Blade Displacement Magnification
$\mathbf{q}$	Vector of Modal Displacements
$\mathbf{f}^E$	Vector of Modal Forcing
$\mathbf{D}$	Modal Damping Matrix
$\mathbf{K}$	Modal Stiffness Matrix
$\mathbf{M}$	Modal Mass Matrix
$\mathbf{Z}$	Aerodynamic Impedance Matrix

ACKNOWLEDGMENTS

The work has been supported by both Kompressorenbau Bannewitz GmbH (KBB) and the Research Association Combustion Engines (FVV). The authors thank for this commitment. The independent investigations contribute to the research project 1389 'Forced response limitation by means of intentional mistuning' (Funding ID: 20992 BG) with a funding by the German Federal Ministry of Economic Affairs and Climate Action (BMWK / AiF).

References

Feiner, D. (2003). A fundamental model of mistuning for system identification and forced response prediction. PhD-Thesis, Pittsburgh: Carnegie Mellon University.

Giersch, T., Hönisch, P., Beirow, B., Kühhorn, A. (2013). Forced Response Analysis of Mistuned Radial Inflow Turbines. *Journal of Turbomachinery*, 135 (3), pp. 031034-1-9. https://doi.org/10.1115/1.4007512

Han, Y., Murthy, R., Mignolet, M. P., Lentz, J. (2014). Optimization of Intentional Mistuning Patterns for the Mitigation of Effects of Random Mistuning. *Journal of Engineering for Gas Turbines and Power*, 136, No. 6, Paper 062505, pp. 1-9. https://doi.org/10.1115/1.4026141

Kühhorn, A. and Beirow, B. (2010). Method for Determining Blade Mistuning on Integrally Manufactured Rotor Wheels. Patent US 2010/0286934 A1.

Lim, S.-H., Castanier, M. P., Pierre, C. (2014). Intentional Mistuning Design Space Reduction Based on Vibration Energy Flow in Bladed Disks. ASME-Paper GT2004-53873. https://doi.org/10.1115/GT2004-53873

Maia, N. (1988). Extraction of valid modal properties from measured data in structural vibrations. Diss. London: Imperial College of Science, Technology and Medicine, University of London.

Nakos, A., Beirow, B. Zobel, A. (2021). Mistuning and Damping of a Radial Turbine Wheel. Part 1: Fundamental Analyses and Design of Intentional Mistuning Pattern. *Journal of Engineering for Gas Turbines and Power*, 18 August, 2021. https://doi.org/10.1115/1.4052201

Nakos, A., Beirow, B., Zobel, A. (2022). Mistuning and Damping of a Radial Turbine Wheel. Part 2: Implementation and Validation of Intentional Mistuning. Proceedings of ASME Turbo Expo 2022, ASME Paper Number: GT2022-80643, Rotterdam, The Netherlands, June 12-17, 2022. https://doi.org/10.1115/GT2022-80643.

Nakos, A., Beirow, B., Wirsum, M., Schafferus, S., Sasakaros, S., Vogt, D., Zobel, A. (2023). Mistuning and Damping of a Radial Turbine Wheel. Part 3: Validation of Intentional Mistuning During Machine Operation. Proceedings of ASME Turbo Expo 2023, ASME Paper Number: GT2023-101993, Boston, Massachusetts, June 26-30, 2023.

Nipkau, J. (2010). Analysis of mistuned blisk vibrations using a surrogate lumped mass model with aerodynamic influences. PhD - Thesis at Brandenburg University of Technology Cottbus.

Sasakaros, S., Schafferus, S., Wirsum, M., Zobel, A., Vogt, D., Nakos, A., Beirow, B. (2023). Experimental Investigation of Synchronous Flow Induced Blade Vibrations on a Radial Turbine - Part 2: Influence of Different Inlet Guide Vane Configurations. Proceedings of ASME Turbo Expo 2023, ASME Paper Number: GT2023-102243, Boston, Massachusetts, June 26-30, 2023.

Schafferus, S., Sasakaros, S., Wirsum, M., Zobel, A., Vogt, D., Nakos, A., Beirow, B. (2023). Experimental Investigation of Synchronous Flow Induced Blade Vibrations on a Radial Turbine - Part 1: Nominal Inlet Guide Vane. Proceedings of ASME Turbo Expo 2023, ASME Paper Number: GT2023-103037, Boston, Massachusetts, June 26-30, 2023.

Spalart, P. R. and Allmaras, S. R. (1992). A One-Equation Turbulence Model for Aerodynamic Flows. AIAA Paper 92-0439. https://doi.org/10.2514/6.1992-439

Yang, M. T. and Griffin, J. H. (2001). A Reduced-Order Model of Mistuning Using a Subset of Nominal System Modes. Journal of Engineering for Gas Turbines and Power, 123, pp. 893-900. https://doi.org/10.1115/1.1385197