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Dynamic response optimization algorithm applied to random geometrically mistuned bladed disks

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ABSTRACT

Unavoidable manufacturing tolerances in bladed disks can lead to geometric mistuning and generate vibration amplification, which can significantly reduce the reliability of the bladed disks. With the development of 3D measurement as well as high-fidelity modeling techniques, it has become convenient to obtain accurate geometric mistuned bladed disk models. However, the blade vibration response is very sensitive to the geometric mistuning parameters, and it is very difficult to optimize the geometric mistuning directly. In this paper, we propose a blade arrangement optimization algorithm to reduce the blade dynamic response amplification caused by the unavoidable geometric mistuning and investigate the influence law of the magnitude of geometric mistuning on the amplification of blade response. We have validated the optimization method by sampling some geometrically mistuned bladed disks through Monte Carlo simulations. The results show that the optimization algorithm can reduce the blade amplitude amplification ratio by more than 5%.

INTRODUCTION

Mistuning in the bladed disk is the difference in the individual sectors due to manufacturing tolerances, material inhomogeneities, in-service wear, and other similar factors, which leads to different modal characteristics in each sectorCastanier and Pierre (2006). Modal deviations resulting from even small levels of mistuning can lead to a significant increase in the maximum amplitude of the bladed disk. The vibration energy is no longer distributed along all blades but is limited to some blades and leads to the occurrence of vibration localization phenomenaYao et al. (2009). Thus, the mistuning of the bladed disk significantly shortens the high circumferential fatigue (HCF) life of the blade disk. Recently there has been an increasing number of research on geometrical mistuned bladed disks. The geometrical mistuned bladed disk model is more accurate than considering only frequency mistuningLiang et al. (2022). Geometric mistuning parameters have a critical impact on the HCF of turbine engines, and it is important to be able to accurately obtain the extreme values of the forced response due to random geometric mistuning parameters and to consider geometric mistuning optimization methods.

In the past, researchers have focused on constructing the effect of geometric mistuning parameters on the modal characteristics or dynamic response. SinhaBhartiya and Sinha (2014) proposed the use of the proper orthogonal decomposition (POD) method for the second-order Taylor series approximation of the blade geometric deviation parameters to evaluate the variation of the mass and stiffness matrices to better characterize the geometric mistuning parameters. Henry et al. Henry et al. (2017) used principal component analysis (PCA) to extract the characteristic features of the geometric mistuning parameters as input to an alternative model to predict the sector modal parameters. The sector modal parameters are coupled by component modal synthesis to obtain the dynamic response of the mistuned blade disc. The authors constructed the relationship between the geometric detuning parameters and the dynamic response through the surrogate model. Beck et al. (Beck, Brown and Kaszynski, 2019; Beck, Brown, Kaszynski, Carper and Gillaugh, 2019) used a Bayesian substitution model to replace the computation of the required sectoral modal parameters in the component modal synthesis approach to reduce the cost of geometric mistuned finite element(FE) model response computation. The Bayesian substitution model effectively predicts the sectoral modal parameters by b different blade geometry parameters and quantifies the uncertainties associated with the geometric mistuning parameters and the modal parameters. The Bayesian surrogate model proposed by the authors is limited to constructing the relationship between the uncertainty of the blade and the modal parameters, for which the dynamic response of the complete mistuned blade disc is still obtained in the physically based ROM. The above approach improves the efficiency of obtaining the modal parameters of the mistuned blade substructure under the FE model of the blade disc considering the geometric mistuning parameters, but the reduced time cost is insignificant compared to the time required for the FE calculation to obtain the dynamic response of the complete blade disc. The use of physics-based ROM to evaluate the effect of geometric mistuning parameters on the dynamic behavior of the blade disc is still very time-consuming Yuan et al. (2017). Many authors have also proposed prediction models for the dynamic response of mistuned bladed disk based on a data-driven approach, with mistuned data as input and dynamic response as output, applied to the lump parametric modelKelly et al. (2021) as well as the FE model considering frequency mistuning(Scarselli and Lecce, 2004; Deng et al., 2020; Liang et al., 2023), respectively. These methods greatly reduce the computational cost of the dynamic response of mistuned-bladed disks and provide a basis for optimization methods.

For a set of mistuned parameters in a mistuned system, there are $N!$ different blade arrangements for the mistuning parameters in the disk, where N is the number of blades, and it is difficult to optimize the vibration amplification phenomenon of the bladed disk by directly controlling the mistuning parameters of the blades. Therefore, we adjust the position of the mistuning blades in the bladed disk to optimize the vibration amplitude amplification, which is simpler than modifying the geometric mistuning parameters. We use the data-driven model of the mistuned bladed disk and the optimization algorithm to seek the extreme value of the response problem. Most of the methods used to optimize the dynamic response of mistuned bladed disks are based on numerical methods. Petrov et al.Petrov and Ewins (2003) proposed a numerical optimization method to search for the worst-case mistuning modes in the vibration response of a mistuned bladed disk to obtain the maximum amplification factor for the given excitation frequency range, and eventually applied the method to a large FE model of mistuned bladed disk. Liao et al.Liao et al. (2010) proposed a combined optimization method based on a genetic algorithm (GA) and sequential quadratic programming (GA-SQP) to optimize the blade arrangement and mistuning parameters to find the maximum forced response of the mistuned bladed disk to determine the worst-case mistuned mode. Pan et al.Pan et al. (2020) proposed an efficient optimization framework for the amplification of the vibration amplitude of the mistuned bladed disk, which establishes the relationship between the blade arrangement and the dynamic response of the mistuned bladed disk through Gaussian process regression and uses GA to find the optimal s mistuned blade arrangement, thus minimizing the effect of the random mistuning of the blade disk.

However, previously proposed methods for the optimization of mistuned blade arrangement are limited by the computational power, usually using a centralized parametric model or a simplified blade disk for the calculation, which cannot evaluate the amplitude amplification capability considering a geometrical mistuned bladed disk. In this paper, a GA and trained neural network model will be used to find the global approximate solution for the amplification of the amplitude of the mistuned bladed disk. The method can understand the extreme vibration response amplification of machined and manufactured bladed disks under unavoidable geometric mistuning.

This paper is outlined below. Section 2 provides an overview of the geometrical mistuned bladed disk vibration response problem and the algorithm for the optimization of the mistuned blade data-driven model and the GA to reduce the vibration amplification phenomenon. In Section 3, the proposed optimization method is demonstrated on an industrial-bladed disk FE model, highlighting some features of our approach. The contribution concludes with some conclusions and a final comment.

METHODOLOGY

The equation of motion for the forced response of the bladed disk with geometric mistuning can be described as

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{F}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the structure mass, damping, and stiffness matrices of the mistuned bladed disk. And the damping matrix $\mathbf{C}$ can be describe as $\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$. $\mathbf{u(t)}$ and $\mathbf{F(t)}$ are the time dependent dynamic displacement and external force. Furthermore, we suppose that the excitation $\mathbf{F(t)}$ is harmonic in time and contains phase variations only between the different blades. The phase of blade m can be followed as

$$\varphi_m=\frac{2\pi(m-1)\Omega}{n} \quad m=1,2,...,n \tag{2}$$

And the excitation f_m with engine order Ω can be represented as

$$f_m=f_0e^{j(\omega t+\varphi_m)} \tag{3}$$

where f_0 and ω are the amplitude and frequency of harmonic excitation, respectively. The forced response of the bladed disk can be assumed as harmonic, and the response of the v-th degree of freedom is expressed as u_v= $\mathbf{A}(v)e^{j\omega t}$, where $\mathbf{A}$ is the complex values of dynamic responses to be solved. Substituting this equation into the equation of motion equation Eqs. (1) to obtain a set of complex algebraic equations, the response of all blades can be expressed as

$$\begin{aligned}\mathbf{A} &= \left(-\omega^2\mathbf{M} + j\omega\mathbf{C} + \mathbf{K}\right)^{-1}\mathbf{f} \\ \mathbf{f} &= f_0\left[e^{j\varphi_1}, \cdots, e^{j\varphi_i}, \cdots, e^{j\varphi_n}, 0, \cdots, 0\right]\end{aligned} \tag{4}$$

We extract the uncertain parameters of each sector in the bladed disk separately. $\mathbf{M}$ and $\mathbf{K}$ can be expressed as $\mathbf{M}^t + \delta\mathbf{M}$ and $\mathbf{K}^t + \delta\mathbf{K}$ respectively, where $\mathbf{M}^t$ and $\mathbf{K}^t$ are the mass and stiffness matrices of the tuned bladed disk which maintains a consistent structure for each sector; $\boldsymbol{\delta}\mathbf{M}$ and $\boldsymbol{\delta}\mathbf{K}$ are variations in the mass and stiffness matrix due to the geometric mistuning parameters of different sectors. Eq. (4) can be expressed as

$$\mathbf{A} = \left(\mathbf{I} + \mathbf{H}^{(t)-1}(\omega)\sum_{i=1}^{n}\boldsymbol{\delta}\mathbf{P}^i\right)^{-1}\mathbf{A}^t \tag{5}$$

where $\mathbf{H}^t(\omega) = -\omega^2\mathbf{M}^t + j\omega\mathbf{C}^t + \mathbf{K}^t$ is the frequency response matrice of the tuned bladed disk, $\mathbf{A}^t = \mathbf{H}^{(t)-1}(\omega)\mathbf{f}$ is the complex values of the responses of the tuned system, $\mathbf{I}$ is a unit matrix. $\boldsymbol{\delta}\mathbf{P} = \sum\limits_{i=1}^{n}\boldsymbol{\delta}\mathbf{P}^i$ is the mistuning parameter matrix containing n geometric mistuned blades. According to Eq. (5), we need to construct the relationship between the geometric mistuning parameter $\boldsymbol{\delta}\mathbf{P}$ and the dynamic response $\mathbf{A}$ through a DNN model as follow:

$$\mathbf{A} = \mathbf{DNN}(\boldsymbol{\delta}\mathbf{P}) \tag{6}$$

As the positions of the mistuning parameters in the tuned system are highly random, $\boldsymbol{\delta}\mathbf{P}$ consists of a large number of different forms. We performed a decoupling operation on $\boldsymbol{\delta}\mathbf{P}$ and then constructed a neural network model using the individual mistuning parameters as follow

$$\tilde{\mathbf{A}}^i_{m,BD} = \left(\mathbf{I} + \mathbf{H}^{(t)-1}(\omega)\delta\mathbf{P}^i\right)^{-1}\mathbf{A}^{i,t}(V_{m,BD}) \tag{7}$$

where $\boldsymbol{\delta}\mathbf{P}^i$ is the matrix of geometric mistuning parameters $\mathbf{p}^i$ generated by the i-th mistuned blade. $V_{m,BD}$ is the vector of degrees of freedom of the boundary nodes of the m-th blade-disk, and $\tilde{\mathbf{A}}^i_{m,BD}$ is the decoupled blade-disk boundary response. $\mathbf{A}^{i,t}$ is the complex-valued response of the tuned system, which is equal to the mode of $\mathbf{A}^t$ in Eq. (5). The relationship between each boundary response and the single mistuning parameter is obtained, and then the actual boundary response is obtained by coupling the decoupled boundary response through a deep neural network $\mathrm{DNN_{dc}}$ as follow

$$\begin{bmatrix}\tilde{\mathbf{A}}^1_{m,BD} \\ \cdots \\ \tilde{\mathbf{A}}^i_{m,BD} \\ \cdots \\ \tilde{\mathbf{A}}^n_{m,BD}\end{bmatrix} = \begin{bmatrix}\left(\mathbf{I} + \mathbf{H}^{(t)-1}(\omega)\delta\mathbf{P}^1\right)^{-1}\mathbf{A}^{1,t}(V_{m,BD}) \\ \cdots \\ \left(\mathbf{I} + \mathbf{H}^{(t)-1}(\omega)\delta\mathbf{P}^i\right)^{-1}\mathbf{A}^{i,t}(V_{m,BD}) \\ \cdots \\ \left(\mathbf{I} + \mathbf{H}^{(t)-1}(\omega)\delta\mathbf{P}^n\right)^{-1}\mathbf{A}^{n,t}(V_{m,BD})\end{bmatrix} \xrightarrow{\mathrm{DNN_{dc}}} \left(\mathbf{I} + \mathbf{H}^{(t)-1}\omega)\sum_{i=1}^{n}\delta\mathbf{P}^i\right)^{-1}\mathbf{A}^t(V_{m,BD}) \tag{8}$$

where $\mathrm{DNN_{dc}}$ is constructed based on the decoupled boundary responses as the input and the coupled boundary responses as the target. Then the vibration response of the blade tip is obtained by the back-propagation process of the mistuned bladeLiang et al. (2023).

The vibration amplification phenomenon of geometrical mistuned systems can be controlled by controlling the geometric mistuning parameters of individual blades or the arrangement of mistuned blades in the mistuned system. In this paper, Monte Carlo Sampling (MCS) of the geometric mistuning parameters is performed to describe the random geometric deviations of the real blades, and the vibration amplification extremes are found by optimizing the arrangement of the mistuned blades. We use the maximum amplitude of the vibration response near the resonant frequency as the optimization parameter to find the best/worst blade arrangement for a mistuned system in a particular mistuned mode based on the data-driven prediction model.

Typically a mistuned bladed disk contains N different geometric mistuning parameters where each mistuning parameter corresponds to one blade and the individual blades in the disk are numbered from 1 to N. Therefore, the geometric random field of the mistuned system with the initial geometrical mistuned blade parameters sequentially arranged can be expressed as $\mathbf{D} = \left\{\mathbf{d}_1{}^T; \ldots; \mathbf{d}_N{}^T\right\}$. For the blade mistuning parameter alignment optimization problem, the variable $\boldsymbol{b}$ is introduced to describe the alignment order, and the geometric field of the mistuned blade disk becomes $\mathbf{D}_{\mathrm{GMS}} = \left\{\mathbf{d}_{b_1}{}^T; \mathbf{d}_{b_2}{}^T; \ldots; \mathbf{d}_{b_N}{}^T\right\}$. The objective function is the maximum displacement of all mistuned blades in the near-resonant frequency range, which can be defined as

$$Q(\boldsymbol{b}) = \max_{\omega\in[\omega_{\min},\omega_{\max}]}\boldsymbol{q}_A(\boldsymbol{b}) \quad Q(\boldsymbol{b}) \to \max \quad Q(\boldsymbol{b}) \to \min \tag{9}$$

where $Q(\boldsymbol{b})$ is denoted as the maximum displacement of the harmonic response of the mistuned bladed disk in the frequency $\omega \in [\omega_{\min}, \omega_{\max}]$ interval when using the blade alignment variable $\boldsymbol{b}$. The optimization variable $\boldsymbol{b} = [b_1, ..., b_i, ..., b_N]$ is an array of integers from 1 to N without repetitions, and b_i denotes the mistuning parameter $\mathbf{d}_{b_i}$ applied to the blade i. The blade arrangement optimization problem finds the worst and best amplification of the mistuned blade vibration amplitude by varying $\boldsymbol{b}$ separately. We solve the above optimization problem using the GA method, a probabilistic global optimization technique inspired by the natural selection process and population genetics theory. GA provides a general architecture for solving complex optimization problems by randomly selecting to obtain populations of individuals to start evolution and by continuously creating new populations by applying genetic operators to individuals in existing populations. The basic genetic operators of GA include selection, crossover, and mutation.

We first randomly generate the n_{intal}-blade assignment array $\boldsymbol{b}$ to form the initial population of mistuned bladed disks with different blade arrangement orders. The random selection of $\boldsymbol{b}$ is made from the uniformly distributed integers in [1, N], and if an integer is selected multiple times, it will be rejected to ensure that each integer appears only once in $\boldsymbol{b}$. Based on the initial population $\boldsymbol{b}$ generated, we obtain n_{intal} mistuned bladed disk with geometrical deviated random fields. Then, the response $Q(\boldsymbol{b})$ of the largest magnitude in each blade in the geometrical mistuned bladed disk is obtained by using a data-driven model. Then n_{select} parents are selected from n_{intal} initial populations for generating new subsets, and the closer the sample computation results are to the objective function the higher the probability of being selected as a parent, where the probability of selecting the jth individual p_j can be expressed as

$$p_j = \frac{\mathrm{Re}Q(b_j)}{\sum\limits_{k=1}^{n_{\text{intal}}} (\mathrm{Re}Q(b_k))} \tag{10}$$

where

$$\mathrm{Re}Q(b_k) = \frac{Q(b_k) - \min\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i))}{\max\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i)) - \min\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i))} \quad \text{if } Q(b) \to \max$$

$$\mathrm{Re}Q(b_k) = \frac{\max\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i)) - Q(b_k)}{\max\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i)) - \min\limits_{i\in[1,n_{\text{intal}}]}(Q(b_i))} \quad \text{if } Q(b) \to \min$$

We then use the partial matching crossover operator (PMX)Zalzala et al. (1997) to generate new individuals by recombining them according to the selected set of parents. In addition, to maintain the diversity of the new individuals, a mutation operation is performed on the new individuals generated by the crossover by randomly selecting two different integers in the individuals to be mutated and swapping the corresponding indexes in the individuals, where the mutation probability is p (p=0.1). Finally, the above steps are repeated until the optimization is completed.

RESULTS AND DISCUSSION

Fig. 1 shows an industrial blade disk with an outer diameter of 254 mm, a material modulus of elasticity of 121GPa, and a density of 4.48 g/cm^3. The FE model of the tuned bladed disk is generated by the cyclic rotation of one sector of the bladed disk and contains 17 sectors, each of which maintains the same structure. The blade portion of the FE model consists of standard linear brick cells (8-node entities) with 4,110 degrees of freedom per sector, and the disc portion uses hybrid brick cells with 4,053 degrees of freedom per sector.

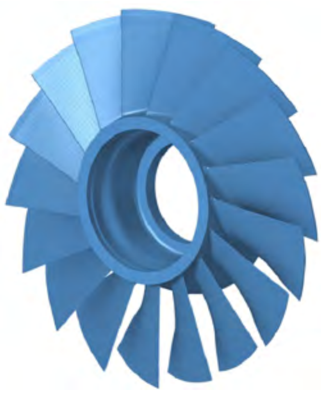

Figure 1 Industrial bladed disk finite element model

The geometric surface of the machined bladed disk was digitized by an optical measurement system with a mean value of 59,469 PCD points per blade as shown in Fig .2 and 3 . The geometric deviation between updated FE model and tuned

FE model shown in Fig.4 was obtained by the mesh deformation methodLiang et al. (2022). Various geometric mistuning is accurately described in the FE model without changing the original structured mesh. The process of generating a high-fidelity FEM from a standard FEM and a measured PCD is fully automated. The histogram in Fig. 5 shows the statistics data of the blade deviations between the tuned FEM and the mistuned FE model. Statistics on blade deviation are used to show the level of geometric mistuning as well as the distribution of mistuning parameters.

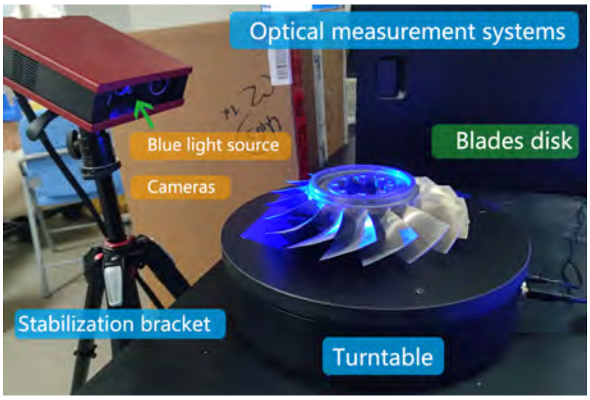

Figure 2 Optical measurement systems

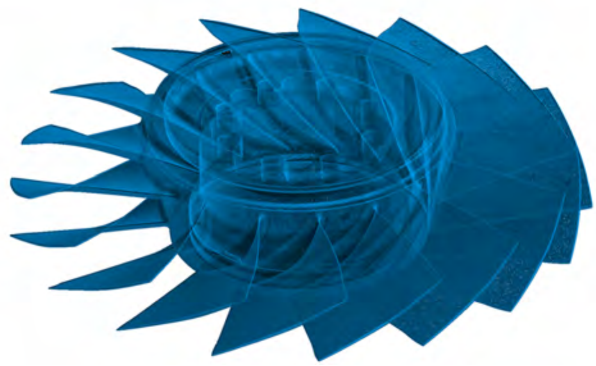

Figure 3 Optical measured data

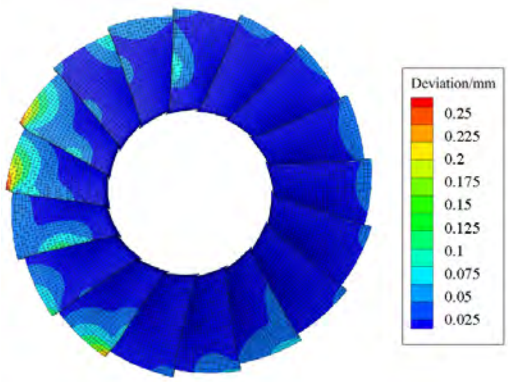

Figure 4 Blade geometric deviation

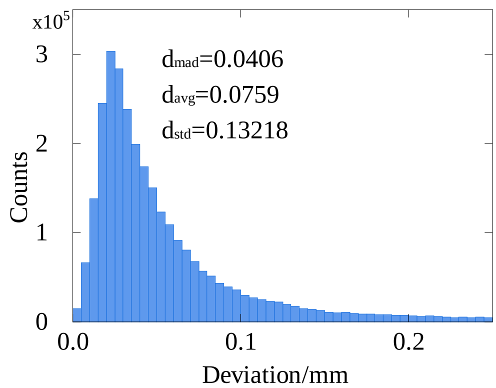

Figure 5 Statistical data of geometric deviations

We used Monte Carlo Sampling (MCS) to obtain 400 geometric mistuning parameters to simulate the measured true geometric deviations. These geometric mistuning parameters are randomly assigned to each blade in the disk to generate 400 random geometric mistuned bladed disk samples. In the harmonic response analysis of mistuned bladed disks, the force amplitude is $f_0 = 5N$, damping ratio $\zeta = 0.0025$, engine order is $\Omega = 1$, and ω is the excitation frequency with an interval of 1 Hz between 1200 and 1250 Hz aim to the first bending mode. The amplification factor (AF) is defined as the ratio of the maximum response in the excitation frequency range of whole mistuned system to the maximum response of tuned system. Figure 6 shows the calculated vibration amplitude response in first modal family of the tuned bladed disk, and the calculated vibration amplitude and frequency envelope of the 400 mistuned bladed disks. The results show that the amplification ratio of the vibration amplitude of the bladed disk under the influence of the random geometric mistuning parameter is 1.18.

Then, we optimized the geometric mistuned blade arrangement using the method introduced in Section 2 to obtain the maximum or minimum amplification ratio of 400 mistuned bladed disks, where the optimization process of the GA algorithm is shown in Fig. 7. Figure 8 shows the distribution of amplitude amplification ratio before and after optimization of 400 geometrical mistuned bladed disk samples. The results show that the optimization method can significantly increase or decrease the amplitude amplification ratio of the random geometric mistuned bladed disk by adjusting the mistuned blade arrangement.

Furthermore, we extract the resonant frequencies of blades in each mistuned bladed disk sample and then calculate the extreme value of blade frequency deviation which can simply reflect the magnitude of the geometric mistuning parameters. In a mistuned bladed disk, there are differences in the intrinsic frequencies of individual mistuned blades in the same mode family. Extreme value of mistuned system frequency deviation represents the difference between the maximum and minimum values of the intrinsic frequency of mistuned blades. Figure 9 shows the relationship between the extreme value of blade frequency deviation and the amplitude factor, which contains a sample of the individual mistuned bladed disk in the MCS set and the optimization results. The results show that the maximum value of the amplification factor gradually

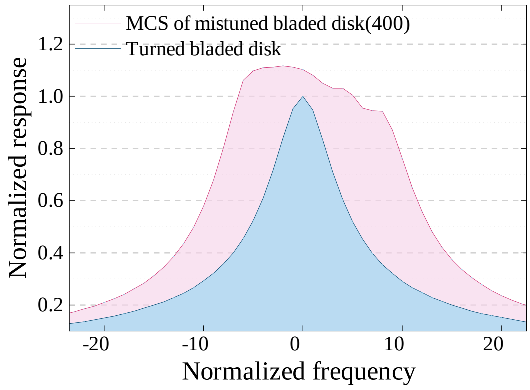

Figure 6 Dynamic response of tuned and mistuned bladed disk obtained by MCS

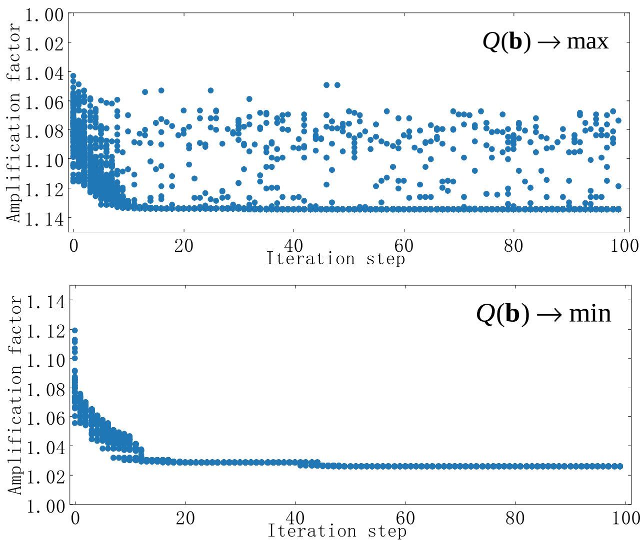

Figure 7 Optimization process of GA

increases as the frequency deviation increase, with no significant effect on the minimum value. Although the MCS obtained by random sampling can reflect the above phenomenon, the extreme values of the amplification factor cannot be accurately obtained, and a relatively conservative estimate will be derived.

CONCLUSIONS

For the geometrical mistuned blade vibration amplification problem, we propose an optimization method based on the blade arrangement order to find the extreme value of the bladed disk amplification. In this paper, the data-driven model is used instead of the FE model for dynamic response calculation to reduce the time component, and the GA optimization method is used to find the global optimal solution for the response amplitude amplification. The results show that the optimized bladed disk has a significantly lower amplitude amplification ratio compared to the MCS sampling results. With the increase of blade mistuning level, the amplification of vibration amplitude of the bladed disks becomes more obvious.

References

Beck, J. A., Brown, J. M., Kaszynski, A. A., Carper, E. B. and Gillaugh, D. L. (2019), 'Geometric Mistuning Reduced-Order Model Development Utilizing Bayesian Surrogate Models for Component Mode Calculations', *Journal of Engineering for Gas Turbines and Power* **141**(10), 101013.
URL: *https://asmedigitalcollection.asme.org/gasturbinespower/article/doi/10.1115/1.4044454/958334/Geometric-Mistuning-ReducedOrder-Model-Development*

Beck, J., Brown, J. M. and Kaszynski, A. (2019), Bayesian Surrogate Modeling of Bladed Disk Sector Mode Shape and Geometrical Spatial Variations, *in* 'AIAA Scitech 2019 Forum', American Institute of Aeronautics and Astronautics, San Diego, California, p. 2002.
URL: *https://arc.aiaa.org/doi/10.2514/6.2019-2002*

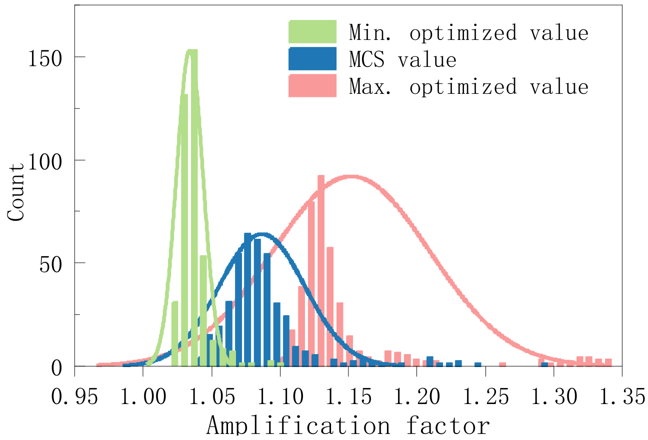

Figure 8 Statistical values of amplification ratio of mistuned bladed disks before and after optimization

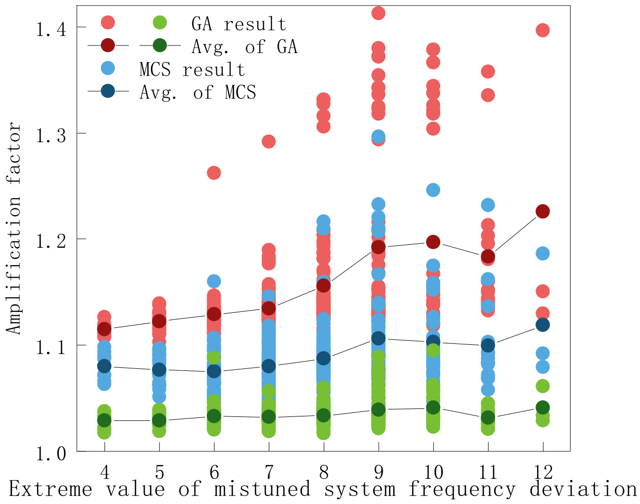

Figure 9 Comparison of amplitude amplification ratio before and after optimization based on mistuning level

Bhartiya, Y. and Sinha, A. (2014), 'Geometric Mistuning Identification of Integrally Bladed Rotors Using Modified Modal Domain Analysis', *J. Eng. Gas Turbines Power* **136**(12), 122504.

Castanier, M. P. and Pierre, C. (2006), 'Modeling and Analysis of Mistuned Bladed Disk Vibration: Current Status and Emerging Directions', *Journal of Propulsion and Power* **22**(2), 384–396.
URL: *https://arc.aiaa.org/doi/10.2514/1.16345*

Deng, S., Yao, J., Wang, L., Xin, J. and Hu, N. (2020), 'Comparative Studies of Surrogate Models for Response Analysis of Mistuned Bladed Disks', *International Journal of Computational Methods* **17**(10), 2050012.
URL: *https://www.worldscientific.com/doi/abs/10.1142/S0219876220500127*

Henry, E. B., Brown, J. M. and Beck, J. A. (2017), Mistuned Rotor Reduced Order Modeling with Surrogate-Modeled Airfoil Substructures, *in* '58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference', American Institute of Aeronautics and Astronautics, Grapevine, Texas.
URL: *https://arc.aiaa.org/doi/10.2514/6.2017-1600*

Kelly, S. T., Lupini, A. and Epureanu, B. I. (2021), 'Data-Driven Modeling Approach for Mistuned Cyclic Structures', *AIAA Journal* **59**(7), 2684–2696.
URL: *https://arc.aiaa.org/doi/10.2514/1.J060117*

Liang, D., Jia, Z., Cao, Z., Wu, Y., Zhang, R., Fan, J., Wu, C. and Yao, J. (2022), 'Geometric mistuning identification and finite element model updating methods for bladed disks', *Aerospace Science and Technology* **130**, 107915.
URL: *https://www.sciencedirect.com/science/article/pii/S1270963822005892*

Liang, D., Yao, J., Jia, Z., Cao, Z., Liu, X. and Jing, X. (2023), 'Novel neural network for predicting the vibration response of mistuned bladed disks', *AIAA Journal* **61**(1), 391–405.
URL: *https://arc.aiaa.org/doi/10.2514/1.J062215*

Liao, H., Wang, J., Yao, J. and Li, Q. (2010), 'Mistuning Forced Response Characteristics Analysis of Mistuned Bladed Disks', *Journal of Engineering for Gas Turbines and Power* **132**(12), 122501.
URL: *https://asmedigitalcollection.asme.org/gasturbinespower/article/doi/10.1115/1.4001054/408825/Mistuning-Forced-Response-Characteristics-Analysis*

Pan, W., Zhang, M. and Tang, G. (2020), 'Blade Arrangement Optimization for Mistuned Bladed Disk Based on Gaussian Process Regression and Genetic Algorithm', *Journal of Engineering for Gas Turbines and Power* **142**(2), 021008.
URL: *https://asmedigitalcollection.asme.org/gasturbinespower/article/doi/10.1115/1.4045553/1070664/Blade-Arrangement-Optimization-for-Mistuned-Bladed*

Petrov, E. P. and Ewins, D. J. (2003), 'Analysis of the Worst Mistuning Patterns in Bladed Disk Assemblies', *Journal of Turbomachinery* **125**(4), 623–631.
URL: *https://asmedigitalcollection.asme.org/turbomachinery/article/125/4/623/449385/Analysis-of-the-Worst-Mistuning-Patterns-in-Bladed*

Scarselli, G. and Lecce, L. (2004), Non Deterministic Approaches for the Evaluation of the Mistune Effects on the Rotor Dynamics, *in* '45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference', American Institute of Aeronautics and Astronautics, Palm Springs, California, p. 1748.
URL: *http://arc.aiaa.org/doi/abs/10.2514/6.2004-1748*

Yao, J., Wang, J. and Li, Q. (2009), 'Robustness Analysis of Mistuned Bladed Disk Using the Upper Bound of Structured Singular Value', *Journal of Engineering for Gas Turbines and Power* **131**(3), 032501.
URL: *https://asmedigitalcollection.asme.org/gasturbinespower/article/131/3/032501/466139/Robustness-Analysis-of-Mistuned-Bladed-Disk-Using*

Yuan, J., Scarpa, F., Allegri, G., Titurus, B., Patsias, S. and Rajasekaran, R. (2017), 'Efficient computational techniques for mistuning analysis of bladed discs: A review', *Mechanical Systems and Signal Processing* **87**, 71–90.
URL: *https://linkinghub.elsevier.com/retrieve/pii/S0888327016303880*

Zalzala, A. M., Fleming, P. J. and Fleming, P. (1997), *Genetic algorithms in engineering systems*, Vol. 55, Iet.