Analysis of Rotating Instability in Axial Compressor Based on Dynamic Mode Decomposition

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ABSTRACT

Rotating instability (RI) is an unsteady flow phenomenon occurring in axial compressors, which is usually accompanied with strong fluctuations and complex structures in the flow field. In this paper, the unsteady numerical simulation of the full-passage Rotor35 model is conducted. A complex, spatially and temporally flow structure of RI is recovered by dynamic mode decomposition (DMD) method. The results show that, DMD is capable of identifying multiple frequency peaks relating to RI, and the dominant mode numbers of circumferential pressure perturbation are 6, 7, 8 and so forth. In addition, the reconstruction results of dominant mode show that the circumferential propagation features of perturbation at different axial positions are different, due to the interaction of RI with the impeller blades. At the position away from the blade, the perturbation propagates circumferentially like a traveling wave; at the position close to the blade, however the response of the blades to perturbation is the main feature, and thus there exist more mode numbers.

1 INTRODUCTION

RI can be considered as pre-stall disturbances, different from modes or spikes. The subject about RI is unlikely to go away if the trend toward smaller engine cores and larger tip clearances continues (Day, 2016). The typical feature of RI is the side-by-side peaks along the hump at about one-third of the blade passing frequency (BPF), which causes the blade vibration and increases the tip clearance noise for axial compressor (Zhu et al., 2018; Brandstetter and Schiffer, 2018; Suzuki et al., 2020).

Many researches have been conducted to investigate the generation mechanism of RI. Mailach conducted an experiment based on a low speed compressor to investigate the RI phenomenon (Mailach et al., 2020). The author attributed RI to the fluctuation of the blade tip vortex. The mode number of circumferential disturbance was about half of the blade number. In März’s research, when RI occurred, a vortex different from the classical tip clearance vortex could be found in the blade tip region (März et al., 2001). The author thought that this vortex was formed by the interaction of the reversed flow in the blade passage, the tip clearance flow and the incoming flow, and that the unsteady behavior of this vortex structure causes the pressure fluctuation associated with RI. Based on the experimental and numerical results, Hah also thought RI was caused by the tip clearance vortex near the leading edge (Hah et al., 2008; Hah, et al., 2010). Li et al. (2021) associated RI with the transverse vortex and the shedding vortex at the rear of the suction side every two passages when tip clearance existed. While without clearance, the mode order of the RI changed from 31 to 10 and the frequency showed comb-like distribution in the tip region. Schrapp et al. (2008) conducted the experimental investigation of the breakdown of the tip clearance vortex both in the linear cascade and the compressor. Based on the PIV measurements, the author proposed that the RI source was the periodic oscillation of the vortex breakdown. Chen et al. (2021) investigated the relationship between the tip leakage flow, tip aerodynamic loading and RI through full-annular simulations. In their
research, RI developed synchronously with the oscillation of the tip leakage flow and blade tip aerodynamic loading, due to the circumferential propagation of disturbance.

Several studies reported that RI was responsible for the significant increase in blade vibration. Vo investigated the relation between RI with the nonsynchronous vibration (Vo, 2010; Thomassin et al., 2011). The author concluded that, when the tip clearance backflow was below the blade tip of trailing edge, the impingement of this backflow on the pressure side at the blade trailing edge could result in the flow oscillations associated with RI. Also, these flow oscillations might be one driver for nonsynchronous vibrations. The fluid-structure coupled simulation of Suzuki et al. (2020) also showed that RI in a compressor of industrial gas turbine with a large tip clearance could lead to nonsynchronous vibration with frequency lock-in due to the influence of RI on the number of disturbance cells and frequency proximity between RI and blade natural mode.

RI was also denoted as a noise source, which even has higher amplitude than the noise relating to blade rotating. Kameier experimentally studied the acoustics noise when RI occurs in a low speed axial fan (Kameier and Neise, 1997a; Kameier and Neise, 1997b). The authors assumed that the noise sources have specific azimuthal pressure distribution and rotate against the rotor with the specific frequency. In Kameier’s research, the frequency differences between the side-by-side peaks were identical and it could be used to calculate the circumferential traveling speed of the noise source. However, based on the experiment of annular cascade and axial fan, Pardowitz found that the frequency differences between the RI peaks were not always identical. Besides, the experiment on a low-speed shrouded-rotor was conducted by Pardowitz et al. (2015) and the RI features had still been clearly found. Therefore, the unsteady behavior of the tip vortex was coupled with RI obviously, but it was not the basic source.

After a literature review on the typical properties and different explanations for RI phenomenon (Pardowitz et al., 2012; Pardowitz et al., 2014; Pardowitz et al. 2015), Pardowitz attributed the flow mechanism of RI source to the instability of shear flow. Particularly, when RI occurs, there exists the inverse flow at the blade leading edge, which forms the condition for the instability of shear flow. Eck assumed that the vortices flow in the RI behave like a characteristic feature of the Kelvin-Helmholtz-Instability (Eck et al., 2017).

In this paper, the stability of the flow field in the Rotor35 is analyzed near stall condition. A complex, spatially and temporally inhomogeneous flow structure about RI will be illustrated by the DMD method. The numerical simulation and results are given in the section 2. Section 3 describes the DMD techniques applied for analyzing the unsteady flow field in this paper. DMD modes are capable of visualizing the flow structures with specific frequencies and further used to analyze the circumferential propagating and distribution characteristics of RI perturbation.

2 METHODOLOGY

2.1 Numerical simulation

NASA Rotor35, a transonic axial compressor rotor, was applied in this paper. It was designed and tested at NASA Lewis Research Center (Reid and Moore, 1978). In designed operation condition, the rotating speed of rotor is 17196.8rev/min. The blade number is 36, and thus the BPF of Rotor35 is 10317.79Hz. Besides, in design operation condition, the total pressure ratio is 1.865, and the mass flow rate is 20.18kg/s. More details of the geometry and performance parameters could refer to the report (Reid and Moore, 1978). Figure 1 presents the full-passage Rotor35 model adopted in this paper.

![Figure 1 The sketch of full-passage Rotor35 model](image)

The single-passage grid was generated by ANSYS-Turbogrid, and the grid independence verification was carried out based on the single-passage model, as shown in Figure 2. In this paper, the grid of 0.42 million was adopted. Then, the
computational domain, the full-passage grid, was obtained by rotating and duplicating the aforementioned single-passage grid. The total grid number of the full-passage model adopted in the paper is 15.12 million.

The numerical simulation was conducted by solving unsteady three-dimensional Reynolds-Averaged Navier-Stokes equations, using the commercial software, ANSYS CFX. The given inlet condition was total temperature and total pressure; the given outlet condition was static pressure. The condition of solid boundary was no-slip and no-heat transfer. The performance parameters under different mass flow rates were obtained by changing the static pressure at the outlet. The turbulence model used in this simulation was k-epsilon turbulence model. In the unsteady simulation, the time interval was $4.846 \times 10^6$ s, which meant 20 physical time step for each blade passage. Three numerical probes, which were shown by the three red points in Figure 1, were set at the 99% span. Besides, these three probes were located at the same circumferential position but three different axial positions (a1, a2, a3), upstream of the blade.

![Figure 1](image1.png)

**Figure 1** The grid independence verification for single-passage Rotor35 model

### 2.2 Dynamic mode decomposition

A simple introduction of dynamic mode decomposition (DMD) is given in this section. DMD is a data-driven method, which was proposed by Schmid (2010). For a series of snapshots, DMD can get the dominant modes, which are associated with the dominant structure in the complex flow. Besides, each DMD mode is oscillating with one single frequency. Therefore, DMD is an effective tool to identify the flow structure and the dynamic flow features, especially for the flow phenomena with typical frequencies. Schmid showed the effectiveness of DMD for the identification of flow structures of an axisymmetric water jet, and this research provided instruction for the application of DMD on analyzing complex flow (Schmid and Pust, 2011; Taira, 2017). Mariappan applied DMD to analyze the unsteady features of the flow field around the pitching airfoil working under attached flow condition and dynamic stall condition, respectively (Mariappan, et al.). The authors also examined the DMD results when the snapshots lack the flow information in the near-wall region in consideration of the difficulty in measuring the flow field in this area by PIV. Lungo proposed a novel prediction model for the wind turbine wake flow based on DMD analysis (Lungo et al., 2015). This new model could exactly capture the dynamic flow features and reduce the computation cost. Hua applied DMD to investigate the cyclic behavior of the stock market (Hua et al. 2016). Some researches focused on the DMD method itself. Differing from the traditional DMD mode evaluation method, Kou (2017) proposed an improved criterion for ranking DMD modes. The proposed mode selection strategy was based on the integration of DMD mode’s time coefficient and performed very well in the test case. Dawson et al.’s research (2016) was about the effect of sensor noise on the performance of DMD. The author not only proposed three modification methods to the standard DMD algorithm but discussed the merits of each modification algorithms.

A simple induction of DMD is as follows. Assume $x_i$ is the snapshot at one time step $i$, which means the parameters of the flow field are included in the snapshot $x_i$, for example, pressure. Then, the snapshot matrix can be written as,

$$X = [x_1, x_2, ..., x_n-1]$$

$$X' = [x_2, x_3, ..., x_n]$$

(1)

where $X, X' \in \mathbb{R}^{m \times n}$. The time interval between two adjacent snapshots is the same and denoted by $\Delta t$. Assume that a linear map $A$ is satisfied between any two consecutive snapshots, which means,

$$x_{i+1} = Ax_i$$

(2)

where $i \in [1, n-1]$. If the snapshots are obtained from a non-linear process, the above linear map means a linear tangent approximation. Based on the above assumption, the snapshots can be written as a Krylov sequence.

$$X' = AX = [Ax_1, Ax_2, ..., Ax_{n-1}] = [Ax_1, A^2x_1, ..., A^{n-1}x_1]$$

(3)
Apply the singular value decomposition (SVD) on $X$,

$$X = U \Sigma V^*$$  \hspace{1cm} (4)

where $U \in \mathbb{C}^{m \times r}$, $\Sigma \in \mathbb{C}^{r \times r}$, $V \in \mathbb{C}^{n \times r}$. Besides, $r$ indicates the rank of matrix $X$, which is equal to the number of singular value, the diagonal elements of matrix $\Sigma$. $V^*$ indicates the conjugate transportation of matrix $V$. To select the dominant singular values, the following strategy can be adopted,

$$\frac{\sigma_i}{\sigma_{\text{max}}} \leq \delta$$  \hspace{1cm} (5)

where $\sigma_i$ is the $i$-th singular value, and $\delta$ is the selection criterion. Then, the matrix $\tilde{A}$, the similarity matrix of $A$, can be obtained,

$$\tilde{A} = U^*AU = U^*XV\Sigma^{-1}.$$  \hspace{1cm} (6)

Obviously, the matrix $\tilde{A}$ has the same eigenvalues with $A$. Therefore, apply the eigenvalue decomposition on matrix $\tilde{A}$,

$$\tilde{A}W = \Lambda W$$  \hspace{1cm} (7)

where $\Lambda$ is the eigenvalue matrix of $\tilde{A}$, and $W$ is the corresponding eigenvector matrix. Therefore, the DMD mode $\Phi$ can be obtained,

$$\Phi = UW$$  \hspace{1cm} (8)

After obtaining the eigenvalues and DMD mode, the growth rate and frequency of DMD mode can be obtained by the real part and imaginary part of $\mu_i$, and $\mu_i$ can be determined as following,

$$\mu_i = \ln(\lambda_i) / \Delta t.$$  \hspace{1cm} (9)

3 RESULTS

The numerical results of single passage model are given and compared with the experimental data in Figure 3. The results show that the total pressure ratio and adiabatic efficiency obtained by numerical simulation overall agree with the experiment well, which proves the reliability of the numerical simulation. Specially, the blue point shows the case adopted in this paper, which is obtained by the numerical simulation of full-passage Rotor35 model.

![Figure 3 The comparison of numerical and experimental results](image)

The pressure fluctuation in rotating reference frame of numerical probes at the red point in the a3 axial position(as shown in Figure 1) in adjacent passages p1, p2 and p3 is shown in Figure 4. The interblade phase angle between different numerical probes is shown clearly, which means the occurrence of RI. The similar features had been found in the previous research about RI conducted by Wu et al. (2016).
As aforementioned, the multiple peaks below BPF (Kameier and Neise, 1997) are the typical features of RI in axial compressor, and thus FFT analysis is conducted for the monitored pressure signal in unsteady numerical simulation. Figure 5 shows the FFT result of the pressure of probe a3 in rotating reference frame at the 99% blade span. The result obviously shows that multiple peaks between 60%-70% BPF exist in this frequency spectrum, which further indicates the occurrence of RI.

To further explore the dynamic features associated with RI, especially the flow structures corresponding to the typical characteristic frequencies of RI, DMD was adopted to analyze the unsteady flow field in the blade-blade surface of the 99% blade span in rotating reference frame. Figure 6 and Figure 7, respectively. Figure 7 shows that there exist multiple DMD modes, of which the frequency falls into the range of characteristic parameters of RI, and the dominant DMD modes associated with RI are marked by green color. The frequencies of these DMD modes are consistent with the characteristic frequencies in FFT result in Figure 5, which proves the reliability and accuracy of DMD method in identifying the dynamic features of unsteady flow. Besides, Figure 6(a) shows that the eigenvalues associated with dominant RI modes are located in the unit circle, which indicates that these RI modes are stable. That is, the perturbation associated with RI is neither increasing nor decaying with time. This conclusion can be confirmed by Figure 6(b), which clearly shows that the real part of eigenvalues associated with RI modes are approximately zero. According to the energy ratio spectrum of DMD mode, the most dominant characteristic frequency of RI is 0.6681BPF, and its second harmonic frequency $f_3$ is also captured, marked by blue color.
3.1 Tip clearance flow characteristics

Numerous researches have indicated that the RI is a part of the complex unsteady flow behavior in tip region. Therefore, the tip clearance flow characteristics are analyzed firstly. Figure 8 shows the streamline sketch of the leakage flow in tip clearance region. For convenient description, the two blades are denoted as B1 and B2, respectively, and the passage between B1 and B2 is denoted as P1. The leakage flow is distributed over the entire chord length, and clearance flow direction is overall perpendicular to the blade chord. At the blade leading edge, the leakage flow, flowing through the tip clearance of the B1 blade, interacts with the incoming flow and forms the leakage vortex in the P1 channel. This leakage vortex is marked with a red circle as shown in Figure 8.

![Figure 8](image)

**Figure 8** The sketch of leakage flow streamline at blade tip region

A similar result can be obtained from the vorticity distribution at different sections overall perpendicular to the blade chord, as shown in Figure 9(a). The vorticity is calculated from the absolute velocity. Since the intensity of the leakage vortex is relatively large at S1–S4, the entire area where the leakage vortex is red. At the S5 section, however, the vorticity contour shows that the center of the leakage vortex is no longer concentrated on the trajectory determined by the leakage flow at S1–S4 sections, indicating the leakage vortex has been broken. Figure 9(b) shows the vorticity distribution at 98% span, which also clearly shows the development and breakdown of the leakage vortex at blade tip region of P1 passage.

![Figure 9](image)

**Figure 9** The sketch of streamline and the distribution of vorticity at blade tip region

Figure 10 shows the Mach number distribution at 98% span. It can be seen that, at the area where the leakage vortex is broken, the Mach number drops sharply (from about 1.5 to 0.5), which indicates that there exists a strong shock. Therefore, the leakage vortex breaks after interacting with the shock wave. In addition, it is worth noting that, the flow in different passages is not exactly the same, but the dominant flow structures and characteristics, the formation and breakdown of the leakage vortex, are similar. The above results are similar to the previous research about the unsteady characteristics of the tip clearance flow under RI conditions of axial compressors by Wu et al. (2016). Downstream of the shock, a blockage zone with low relative velocity develops. The trajectory of the clearance vortex moves in the upstream direction. The reversed clearance flow and associated blockage enables the form of the instability wave from the view of the flow stability (Pardowitz et al., 2014; Eck, 2017).
3.2 The DMD modes

In this section, several dominant DMD modes are showed to analyze the typical features associated with RI. Figure 11 shows four dominant DMD modes, of which the frequencies are 0.6377BPF, 0.6529BPF, 0.6681BPF and 1.3190BPF, respectively. Note that the display range is only about 8 passages in circumferential direction and entire axial range.

![Figure 11 The four dominant DMD modes](image)

To show the perturbation distribution in the upstream region of the blade more clearly, the three dominant RI modes in this region (0.6377BPF, 0.6529BPF and 0.6681BPF) are shown in Figure 12(a). Besides, Figure 12(b) shows the corresponding value at a3 axial position obtained from DMD mode. In Figure 12(b), the horizontal coordinate axis indicates the circumferential position, and thus this result shows the circumferential distribution of relative perturbation. It is clearly shown that the dominant mode numbers of circumferential pressure perturbation at a3 are different for different DMD modes, and they are 6, 7 and 8 respectively for these three RI modes.
3.3 The characteristic parameters of circumferential rotating waves

To precisely compare the mode numbers of RI perturbation at different axial position, the FFT is applied on the corresponding values extracted from DMD mode. Take the most dominant RI mode (0.6681BPF) for example. Figure 13 shows the extracted values at a1, a2 and a3 axial position, and they are marked by red, green and blue, respectively.

Then, apply the FFT analysis for them and the results are shown in Figure 14. The results show the dominant mode numbers at different axial position are different. At a3 position, which is away from the blade, the dominant mode number
is 8; however, at a1 position, near the blade, the dominant mode number includes not only 8 but 26, 44, which are the results of the subtraction or addition of 8 and the blade number (36). Therefore, the mode number of the rotating waves near the blade can be calculated by the following equation,

\[ k = k_{in} \pm nN_b, \]

where \( k_{in} \) indicates the dominant mode number of the rotating waves away from the blade. Besides, \( n \) is a non-negative integer; and \( N_b \) is the blade number of Rotor35 (36).

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The data are still needed to investigate the generation mechanism of RI and analyze the RI characteristics in axial compressor. Temporally, the flow structure of RI is similar to a traveling wave; at the position near the blade, however, the perturbation mainly shows the blade’s response to the upstream perturbation, and thus owns more mode numbers. Reduced domain models with period boundary conditions artificially fix the wave number and propagation speed and may give misleading results. The complex, spatially and temporally varying structure of RI perturbation is illustrated at different circumferential positions. The stability of the flow field is close to the flow mechanism of RI source. However, it is worth mentioning that more data are still needed to investigate the generation mechanism of RI.

4 CONCLUSION

In this paper, the numerical simulation of full-circle Rotor35 was carried out, and DMD was applied to analyze the dynamic characteristics associated with RI of axial compressor. The result indicates that, DMD is capable of capturing multi-order modes associated with RI. Ranked by the energy ratio, the frequency of the dominant RI mode is 0.6681BPF with the circumferential mode number 8. Along the dominant RI modes, other circumferential modes 7 and 6 are also shown. Besides, the circumferential distribution and propagating characteristics of the RI perturbation are illustrated at different axial positions. At the position away from the blade, the perturbation associated with RI exhibits the characteristic similar to a traveling wave; at the position near the blade, however, the perturbation mainly shows the blade’s response to the upstream perturbation, and thus owns more mode numbers. Reduced domain models with period boundary conditions artificially fix the wave number and propagation speed and may give misleading results. The complex, spatially and temporally varying structure of RI is a part of the unsteady flow in the tip region, and DMD is an effective tool to illustrate and analyze the RI characteristics in axial compressor.

The stability of the flow field is close to the flow mechanism of RI source. However, it is worth mentioning that more data are still needed to investigate the generation mechanism of RI.

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