Analysis On The Spike-type Rotating Stall For Axial Compressor By Dynamic Mode Decomposition

Moru Song
Shanghai Jiao Tong University
sumner0624@163.com
Shanghai, China

Bo Yang
Shanghai Jiao Tong University
byang0626@sjtu.edu.cn
Shanghai, China

Guoming Zhu
Shanghai Jiao Tong University
1562161048@qq.com
Shanghai, China

Shui Zhang
Shanghai Jiao Tong University
homedown.good@163.com
Shanghai, China

Hong Xie
Shanghai Jiao Tong University
369165499@qq.com
Shanghai, China

ABSTRACT
In this paper, the spike-type rotating stall for axial compressor have been studied by dynamic mode decomposition. The full-wheel model with unsteady Navier-Stokes simulation is established and the atmospheric boundary condition is applied in order to obtain the detailed unsteady flowfield. The Omega-criterion is applied for better visualization of vortex. The stall process, including stall inception and stable stall, has been captured and studied. It is found the fluctuation in the compressor flow is dominated by blade passing in the stall inception stage, while it is dominated by stall cell in the stable stall stage. The stall onset is triggered by the increasing of the momentum ratio between tip leakage flow and main flow, developed by the adverse pressure gradient between the downstream and upstream, and finally sustained in a stable manner. The stall cell consists of a pair of high-low pressure cells. The unsteady results with high nonlinearity were then decomposed by the dynamic mode decomposition (DMD). By analyzing the modes and the reconstruction results, the leading characteristics during the stall inception and the fully-development are dug out. The stage of the stall inception and steady stall can be easily recognized by the mode frequency and energy. The frequency and the strength of the flow structure can be reflected by the corresponding mode shape.

INTRODUCTION
Rotating stall, as one of the most destructive issues for an axial compressor, must be prevented during the operating of the machine. Over the decades of researches on compressor, how to precisely predict, deeply understand and actively control the rotating stall has been one of the most challenging topics due to the highly-nonlinearity, highly-instability and the limited-measurement with much danger.

There are mainly two types of the stall inceptions: the modal-type with long length-scale wave and the spike-type with small disturbance. The latter one is more undesirable, because it is not easy to obtain a satisfied result when the analysis and control methods were applied to the abrupt-formation of the spike-type disturbance. Up to now, the mechanism for the evolution of the spike-type stall is understood mainly by temporal analysis. Cumpsty(Cumpsty, 1989b) and Hoying(Hoying et al., 1998) related the spike-type stall inception to the tip leakage flow (TLF) and the correspond tip clearance vortex (TCV). When the stall inception occurred, it was found the TLF and the TCV are spilled out of the flow passage forwardly and flowed into the neighboring blade passage. Vo(Vo et al., 2008) concluded that the forward and rearward spillage of the tip leakage flow can be the criteria for the prediction of the spike-type stall inception. However, Inoue(Inoue et al., 2000) found out the radial vorticity in front of the flow passage is crucial to the formation of the spike-type stall, but the radial vorticity is not the spillage of TCV. Based on the numerical work for the 2D cascade and 3D full-annulus axial compressor, Pullan(Pullan et al., 2015) and Smith(Hewkin-Smith et al., 2019) found the spike-type stall can be occurred in the model without tip clearance, and the radial vorticity was found as a key role on the formation of the stall. In their
model, the radial vorticity in front of the passage is generated by the leading-edge separation under the high-incidence inflow instead of TLF or TCV. Ning(Ju and Ning, 2014) obtained the numerical result of the pre-stall condition for NASA rotor67. Although the flow spillage was found in front of the passage, there is no radial vorticity in the corresponding region at all. So far, the pattern of the spike-stall inception is not clear enough.

Meanwhile, many mathematical methods were introduced in order to dig out more information from the dataset of the spike-type stall. Fourier analysis, as one of the most comprehensive method, was always applied to the periodic unsteady pressure/velocity data in order to find out the character frequencies and then the corresponding underlying flow patten. For example, the periodic fluctuation by the rotor-stator interaction, the blade passing or the stall cell rotating can be easily recognized by the stall cell. However, the Fourier analysis is not a globe and multi-dimensional method, thus the field data is hard to be directly analyzed by it(Pochylý et al., 2019). Dynamic mode decomposition (DMD) (Schmid et al., 2011), as one of the reduction-order method, have the both advantages of the frequency-analysis and the globe analysis. Thus, the flows in the turbomachinery are suitable for the DMD method(Rowley and Dawson, 2017, Lengani et al., 2016, Mariappan et al., 2014), such as the flow in the centrifugal compressor(Li et al., 2021), axial turbine(Romero and Gross, 2019) and the axial compressor(Song and Yang, 2021).

In addition, the numerical study on the compressor stall is still limited by numerical resources and computational. Theorically, as the circumferential-propagated stall cell could be captured only by full-wheel grid, tens of millions of grid must be generated for every row, which is hardly afforded. Not only that, the common outlet boundary does not work for the stall prediction even using radial equilibrium treatment because the real unsteady fluctuation at outlet can not be specified correctly. Cumpsty(Cumpsty, 1989a) and Vahtadi(Vahtadi et al., 2004) put forward a treatment that a nozzle model can be added behind the outlet boundary. Recently, this treatment has been successfully applied on researches(Zhao et al., 2018, Zhang and Vahtadi, 2019) related to axial compressor stall. These researches gives a confidence to obtain stall-related flowfield from numerical approach.

In this paper, the unsteady flowfield during stall onset and stable stall is obtained by the numerical simulation. The mechanism for the generation and propagation of the stall cell is firstly understood by the temporal analysis. Then, the unsteady datasets under different levels are decomposed into DMD modes. By studying the frequency, growth rate and mode shape, the underlying physical significance for every mode is revealed.

**METHODOLOGY**

**Numerical simulation method**

The CFD solver is a commercial code NUMECA, which is based on the 3D compressible viscous RANS equations (1).

\[
\frac{\partial}{\partial t}\int_{\Omega} Ud\Omega + \int_{S} F_{i}d\vec{S} + \int_{S} \vec{F}_{i}d\vec{S} = \int_{S} d\Omega
\]

where \(\Omega\) is the integral volume, and \(S\) is the integral surface. \(U\) is the matrix of conservative variables, which is

\[
U = \begin{bmatrix} \rho & \rho u & \rho w & \rho \rho \end{bmatrix} \tag{2}
\]

where \(\rho\) is density, \(w\) is relative velocity, subscript 1, 2, and 3 are represent for x, y and z direction, respectively. \(E_{t}\) is total energy per mass flow.

Considering that the research compressor is of transonic condition, the JST scheme is adopted in the spatial discretization. In the unsteady simulation, the dual time stepping method is used. The turbulence model is Spalart-Allmaras(SPALART and ALLMARAS), and the flowfield is calculated in the relative frame. As to the boundary condition, the total quantities in the inlet and the atmosphere pressure in the outlet are applied.

To obtain the performance map, the nozzle area is adjusted in every condition. The steady simulation begins with the straight nozzle model with a uniform initial field. When the previous calculation of the straight nozzle model is convergenced, its results will be used as the initial condition in the next steady simulation. In the next simulation, the nozzle area is reduced by 1%~5% of the total area. After several simulations, the near-stall point (NS) will be found out when the steady simulation with a reduced-nozzle model is no longer converged. Then, in the followed unsteady simulation, the nozzle area is still reduced and the previous steady/unsatety result is imported as the initial condition. By this way, the whole process of the stall can be obtained.

**Dynamic mode decomposition method**

DMD is a method that decomposes the unsteady flowfield into modes, with each mode having a single characteristic frequency of the perturbation and growth rate. The compressible Naiver-Stokes system, which governs the flowfield in the compressor, can be described as (3):

\[
\frac{dx}{dt} = f(x,t) \tag{3}
\]
where \( x(t) \in \mathbb{R}^n \) is the state of the dynamic system at time \( t \), such as pressure and velocity field. \( f(\cdot) \) is the right-hand-side of governing equations, which represents the dynamics of the flow system.

The analogous discrete-time system can be described as (4).

\[
x_{k+1} = A(k \Delta t)x_k
\]

where \( \Delta t \) is the sampling period. \( A \) is the matrix that maps k-th flowfield to the flowfield in next timestep. Because of the nonlinear of the system, \( A \) is a non-linear operator and varied with time \( t \).

The idea of DMD is trying to find a best-fit linear operator \( A \) that approximates the nonlinear dynamic operator \( A \) from the unsteady flowfield, then find the modes (eigenvectors of \( A \)) with corresponding frequency and growth rate (eigenvalues of \( A \)). Mathematically, two datasets \( X \) and \( X' \) will be created from the original unsteady flowfield \( X_o \in [x_1, x_2, \ldots, x_m] \):

\[
X \in [x_1, x_2, \ldots, x_{m-1}]
X' \in [x_1, x_2, \ldots, x_m]
\]

The two datasets are related by \( A \):

\[
X' \approx AX
\]

Then the best-fit \( A \) is obtained by

\[
A = X'X'
\]

where \( X' \) is the Moore-Penrose pseudoinverse.

In order to calculate the eigenvector and the eigenvalues of \( A \) efficiently, \( X \) is decomposed by reduced SVD:

\[
X = U_i \Sigma_i V_i^T
\]

where \( r \) is the truncation number. Then let

\[
\tilde{A} = U_i^T A U_i = U_i^T X V_i^T \Sigma_i^{-1}
\]

and find the eigenvectors \( \tilde{v}_j \) and eigenvalues \( \mu_j \) of \( \tilde{A} \). \( \mu_j \) is also the eigenvalues of \( A \). The eigenvectors of \( A \) is:

\[
v_j = \mu_j^{-1} \Sigma_i^{-1} \tilde{v}_j
\]

The growth rate and the frequency for every mode can be obtained from the real and imaginary part of \( \mu_j \).

Research Model and Simulation Domain

The research object is NASA Rotor67, which is a transonic axial rotor as the standard open-model. The detailed experiment data was obtained in NASA Lewis research center(Strazisar et al., 1989). Some key design parameters are listed in Table 1. The geometry and the aerodynamic survey locations are represented in Fig.1(a) and Fig.1(b). The configuration of the calculation domain is illustrated in Fig.1(c). Specifically, the upstream channel and downstream channel are extended as much as 5 times and 8 times of the rotor tip chord, respectively. Refer to the related researches(Cumpsty, 1989a, Vahdati et al., 2004, Choi and Vahdati, 2011), a nozzle is set behind the downstream in order to obtain the in-stall results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade number</td>
<td>22</td>
</tr>
<tr>
<td>Design rotating speed</td>
<td>16,043 (rpm)</td>
</tr>
<tr>
<td>Design tip clearance</td>
<td>1.01 (mm)</td>
</tr>
<tr>
<td>Tip chord</td>
<td>91.8 (mm)</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>1.56</td>
</tr>
<tr>
<td>Tip solidity</td>
<td>1.29</td>
</tr>
<tr>
<td>Design mass flow rate</td>
<td>33.25 (kg/s)</td>
</tr>
<tr>
<td>Design total pressure ratio</td>
<td>1.63</td>
</tr>
<tr>
<td>Tip relative Mach number</td>
<td>1.38</td>
</tr>
<tr>
<td>Inflow Reynolds number</td>
<td>2.7×10^6</td>
</tr>
</tbody>
</table>
Numerical Setup

The full-annulus structured grid is generated by AutoGrid5. The grid is of high quality and the $y^+$ is less than 1. In the region of the tip clearance, the refinement treatment is applied so that the high-unsteady flow can be captured better in this region. In order to ensure that the grid size is large enough, 5 sets of grids are established and the parameters are listed in Table 2. The tip leakage flow, as one of the most sensitive quantities on the grid size, is chosen as the index for the grid independence. From the comparison in Fig.2, the grid3 with the grid number of 38.7 million is chosen for the following calculation.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Rotor</th>
<th>Rotor tip</th>
<th>Full-annulus total(million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid 1</td>
<td>65\times57\times117</td>
<td>65\times17\times117</td>
<td>12.2</td>
</tr>
<tr>
<td>Grid 2</td>
<td>97\times57\times173</td>
<td>97\times33\times173</td>
<td>33.2</td>
</tr>
<tr>
<td>Grid 3</td>
<td>113\times57\times173</td>
<td>113\times33\times173</td>
<td>38.7</td>
</tr>
<tr>
<td>Grid 4</td>
<td>129\times57\times225</td>
<td>129\times33\times225</td>
<td>57.5</td>
</tr>
<tr>
<td>Grid 5</td>
<td>137\times57\times277</td>
<td>137\times33\times277</td>
<td>75.1</td>
</tr>
</tbody>
</table>

Validation of Numerical Simulation

The comparisons between numerical results and experimental data are shown in Fig.3 and Fig.4. From the overall performance map in Fig.3, the red squares represent experimental data, and the blue circle represents the time-averaged numerical result. The experimental and simulation performances are consistent. Not only that, by the unsteady simulation, the flowfield under the stall condition is also obtained. From the curves in the blackbox in Fig.3, the performance under stall condition is of high unsteady and usntable, which is also consistent with the results from the relevant researches. The followed figures in Fig.4 are the isoline of relative Mach number at 90%, 70% and 30% span from hub. From the comparison under near peak efficiency (NP) and near stall (NS) conditions, the distributions obtained from simulation are very close to these from experiment.
TEMPORAL CHARACTERISTICS OF THE ROTATING STALL

In order to detect the fluctuation of the flowfield quantitatively, the numerical probe in the rotating frame is set in front of LE for every passage. Fig.5(a) shows the unsteady pressure from the probe and the corresponding wavelet transform result, in which much information is dug out. Based on the wave shape of pressure-time plot and the time-frequency characteristic, the time histories can be divided into two stages: stall inception \((t < 15 \, \text{revs})\) and stable stall \((t > 15 \, \text{revs})\). In the first stage, the fluctuation is dominated by the blade passing frequency (BPFs), which indicates the unsteady flow is disturbed by the interaction between rotor blade and stationary boundary. Meanwhile, the amplitude of 1/2 BPF, which is highly related with the self-induced unsteadiness of the TCV, is also take advantage in the time-frequency result. Nevertheless, following the blue dot-line, the small distribution is found traveling along the same pitchwise direction as the rotation of the blade. In the second stage, the fluctuation is governed by the traveling of the stall cell. The period of the stall cell wave is 2.65 rev in the rotating frame; thus, the rotating speed of the stall cell is 75% of the rotating speed in the absolute frame, which is consistent with the result from other researches about the stall of Rotor 67. It is also found the large amplitude at around 0.2 BPF, which will be decomposed and discussed in the following section.

In Fig.5(b), the unsteady pressure-field during the stall cell is illustrated by the 10 snapshots of pressure at 99% span and the corresponding pressure distribution chart along LE/TE line. From rev6 to rev8, the perturbation is so small that it is difficult to find the phenomenon out. Meanwhile, the pressure charts along LE/TE shows the periodic spatial fluctuation in every passage. Then, from rev9 to rev10, although there is still hard to find stall signal in the contour, the pressure chart changes with the multi crest inner one passage and the pressure rise. After one more revolution, the low-pressure cell is bursted into and covers for 2–3 passages. In rev12–rev13, the stable stall cell is formed in front of LE. The stall cell is
consist of a high-low pressure cell pair. In this relative frame, the stall cell is sustained and revolved in count-rotating direction. At the same time, through the fluctuation amplitude in the pressure chart, it is found the flowfield in other passages is more stable than that before stall inception.

(a) The pressure signal from numerical probe and the corresponding wavelet result

(b) The contours at 99% span and the distribution along LE/TE of of pressure coefficient (CP)

Fig.5 The unsteady flowfield under the stall condition

In order to study the detailed stall cell evolution from the stall inception to the steady stall, the Omega-criterion, as one of the novel and effective vortex criterions, is introduced for better visualization. In Fig.6, the flowfield is zoomed out to the nearfield of the stall cell, and the streamline of the relative velocity $W$ is added. The stalled region has been labelled by the white-dot box. In rev10, there is a straight vortex stripe in front of LE. Refer to the relevant studies, the vortex stripe is actually the TCV. In stable operation conditions, TCV is always generated near LE and moves downstream along the flow passage. However, during the stall inception, as the mass flow decreases and the pressure ratio increases, the momentum ratio between TLF and main flow (MF) is increased, thus the TCV is pushed out the flow passage by the effect of the TLF. Meanwhile, the backflow appears near the downstream of the stalled passages, because the adverse-pressure gradient can not be sustained in these stalled passages. The backflow also contributes the momentum to the TLF-MF momentum ratio. So, based on this positive-feedback pattern, the stall cell is increased rapidly from rev10 to rev13, until the upstream pressure in these stalled passages has been highly increased to balance with the downstream pressure. The increased pressure in upstream accelerates the flow into the neighbor passage to weaken the blockage, which give rise to the pressure drop of the stall cell. The fully-developed stall cell will occupy 6~7 passages.
MODE ANALYSIS FOR THE ROTATING STALL

Before studying the decomposition results, it is necessary to figure out how many modes that should be decomposed which can capture the most of the information in the original flowfield. Fig. 7 is the root-mean-square error (RMSE) of the reconstruction flowfields from the first $r$-th modes. The RMSE is defined in Eq. (11). By using the first 200 modes, the error of the reconstruction flowfield is less than 6%, which can be accepted for the followed analysis.

$$\text{RMSE}(r) = \frac{\sum_{i=1}^{\text{size}(x)} (x_{\text{raw}} - x_{\text{dmd}})^2}{\text{size}(x)}$$  \hspace{1cm} (11)

where $x_{\text{raw}}$ is the original flowfield data, $x_{\text{dmd}}$ is the reconstruction flowfield data by using $r$ modes, size(x) is the grid points of the flowfield.

Fig. 7 The root-mean-square error (RMSE) of the reconstruction flowfields from the first $r$-th modes
So far, the stall inception characteristics have been studied by analyzing the time-sequential local flowfield. Although we can learn the evolution process of the stall cell, but from the global point of view, it is hard to use this analysis method to answer the questions why the stall cell generated, can the stall cell sustain its shape or what kind of flow structure contributes to the generation of the stall cell. By the means of DMD method, the questions might be studied deeply by evaluating the stability of the dynamic system and extracting the important modes with corresponding frequencies. Theoretically, the dataset $X$ has better being the stationary process. So, referring to the idea of short-time Fourier transform, 4-level time-window is applied to the non-stationary pressure from rev1~rev16. In Fig.8, the abscissa and vertical ordinate represents the frequency and the energy ratio of the DMD modes. The purple and yellow filled in circle means the stable and unstable mode, respectively. The yellow circle in the scatter of 1~16rev indicates the stage of the stall inception is unstable. However, if the time-windows is divided into level 3 or level 4, the modes under first and last several revs are all stable. So, the stall inception is a transitional stage between stable low flow-rate condition to stable stall condition. On the other hand, the energy ratio of zero-order mode is decreased from 1rev to 16rev, which means the stable stall condition is with higher unsteadiness than stable low flow-rate condition.

In order to study the flowfield characteristics by the mode shape, the mode contours in level 3 are shown in Fig.9. 5 modes for every dataset are put into the figures, including one steady mode, stable modes(smode) and unstable modes(umode). It is obvious that the mean-flow, BPF-dominated flow and stall-cell-related flow are identified by the DMD modes. There is a little pitchwise nonuniform from dataset1 and dataset2, while totally uniform from dataset3 and 4, which indicates the evolution from the stall inception to steady stall. Although the stall cell rotates and disturbs the flowfield, its time-averaged effect is limited, which indicates the distributes from stall cell in every passage is becoming identical and periodic as the stall cell grows up. In the stage rev6~rev9, the effect of BPF dominates the flowfield (smode1, smode2), but the perturbation is only occurred inner the passage, which means the strong effect of the stall cell. The other two modes are related to the stall cell, including a unstable mode. In the stage of dataset2~4, the smode1 from dataset2~4 are all the mode of stall cell, while the other two following modes are the harmonic of the smode1. The unstable modes are all related to the generation of the stall cell in the stall inception process.

![Fig.8 The frequency and mode energy for every mode obtained by four-level datasets](image-url)
CONCLUSIONS

In this paper, the detailed flowfields from stall inception to stable stall are obtained by the numerical simulation. By the means of temporal analysis and dynamic mode decomposition, several conclusions are drawn as follows.

1) By the means of the atmospheric outlet condition, the complete flowfield from stall onset to stable stall is obtained with the full-wheel grid and unsteady Naiver-Stokes simulation. It is found the nozzle outlet boundary is critical to the stall inception in the numerical simulation.

2) The characteristic of the flowfield under stall has been studied. The fluctuation in the compressor flow is dominated by blade passing in the stall inception stage, while it is dominated by stall cell in the stable stall stage. The stall onset is triggered by increasing of the momentum ratio between tip leakage flow and main flow, developed by the adverse pressure gradient between the downstream and upstream, and finally sustained in a stable manner. The stall cell sustains with a pair of high-low pressure cells.

3) The unsteady results with high nonlinearity were then decomposed by the dynamic mode decomposition (DMD). By analyzing the modes and the reconstruction results, the leading characteristics during the stall inception and the fully-development are dug out. The stage of the stall inception and steady stall can be easily recognized by the mode frequency and energy. The frequency and the strength of the flow structure can be reflected by the corresponding mode shape.

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