ABSTRACT

Small wind turbines often have to operate in slow and highly turbulent wind. Whereas the aerodynamic design of rotor blades for small wind turbines is mostly based on tools and methods developed for large wind turbines, these aerodynamic requirements distinguish small wind turbines from larger models significantly. Nonetheless, steady blade element momentum (BEM) theory is employed to calculate the rotor aerodynamics, since this theory delivers reasonable results within a fast calculation time. Usually, steady state simulations - which require much less computational effort than unsteady simulations - tend to be preferred during the iterative design process of new rotor blades. This paper explores the worthiness of using computationally taxing unsteady flow simulations for determining the optimum rotor blade shape. A differential evolution algorithm in combination with an unsteady blade element momentum model is applied to derive an optimised blade shape for a small horizontal-axis wind turbine under turbulent inflow conditions. The results of this paper present, compared to steady optimisation, the effect of turbulent inflow on the optimum aerodynamic rotor blade shape and the rotor performance of a small wind turbine.

INTRODUCTION

Knowledge of the aerodynamics of small wind turbines (SWT), and especially the aerodynamic design of the rotor blades for SWT is not as advanced as for larger wind turbines, as e.g. van Treuren (2019) has shown. Despite these discrepancies between large and small wind turbines, often the same design tools and procedures are used for all scales of wind turbines. Differences in aerodynamics arise, for example, due to the low hub heights, different Reynolds numbers, and faster rotational speeds. SWTs often have to operate deep in the atmospheric boundary layer, which leads to lower mean wind speeds and greater fluctuations in wind speed due to very high turbulence intensities. Values of the turbulence intensity of 30% or more are not uncommon in the inflow of small wind turbines.

The necessity of considering the smaller wind speeds during the design process of rotor blades for SWTs has already been recognised, as recent publications show, for example from Hassanzadeh et al. (2016), Mayeed and Khalid (2016), Muhsen et al. (2020) and Tenghiri et al. (2018). Furthermore, Clifton-Smith and Wood (2007) and Wood (2011) developed a methodology to optimise not only the power coefficient, but also the starting behaviour. This is particularly important for SWTs to improve the annual energy production (AEP), as they are usually not able to adjust the pitch of the rotor blades.

All these publications neglect that the wind at these low wind speeds has significantly higher turbulence intensity, as it is for example defined by the standard for SWTs, by the International Electrotechnical Commission et al. (2013). For example at a mean wind speed of 5 m/s the turbulence intensity is at 30% and it is even higher for lower wind speeds. The current design processes used for SWT rotor blades conducts only steady-state simulations which does not take into account the unsteadiness of the incoming wind due to the increased turbulence intensity.

The goal of this paper is to investigate the influence of the large turbulent fluctuations on the design of a SWT rotor blade under low wind speeds using common design tools. Therefore, a numerical design process is employed witch is capable to conduct both steady and the computational higher taxing unsteady blade element momentum (BEM) simulations during an example design process using the NREL Small Wind Research Turbine (SWRT) described by Corbus and Meadors (2005). The resulting blade design and its performance on the SWRT is then compared to a steady blade optimisation.

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METHODOLOGY

Design specifications

The design process uses the SWRT as described by Corbus and Meadors (2005); this wind turbine has been thoroughly investigated, both experimentally and numerically, and is well documented. It is a 10 kW, horizontal axis wind turbine with three rotor blades. The original rotor blades have a constant chord and a constant twist distribution and a single profile is used over most of the rotor blade span. The SH3052 profile is used, which has a thickness of about 13.5% and a camber of 7%. The optimum angle of attack (angle of attack with the highest lift-to-drag ratio) is 4° with a lift coefficient of $C_{lift} = 1.3608$ and a drag coefficient of $C_{drag} = 0.0314$.

This study is intended to redesign the SWRT for a new site, hence, the main characteristics of the wind turbine are the same, i.e. number of blades (3), blade profile (SH3052), tip speed ratio (6) and rotor diameter (2.9 m). The tower height at the new site is limited and a hub height of 12 m has been set in the turbine model. At this height a mean wind speed of 5 m/s is expected, resulting in a turbulence intensity of 30% according to the standard for small wind turbines (International Electrotechnical Commission et al., 2013).

Analytical optimum

As a means of validation an analytical approach has been derived from the explanations of Hansen (2015) and Burton et al. (2001) and the equations used are given in the following:

$$16a^3 - 24a^2 + (9 - 3x^2)a - 1 + x^2 = 0 \quad (1)$$

with:

$$x = \frac{\omega r}{V_0} \quad (2)$$

The optimum axial induction factor, $a$, depends on the local radial position and can be calculated by solving equation (1) where $x$ is the local tip speed ratio, $\omega$ the rotational speed of the rotor, $V_0$ the wind speed and $r$ the radial position. The optimum tangential induction factor, $a'$, is then defined in terms of $a$ by equation (3).

$$a' = \frac{1 - 3a}{4a - 1} \quad (3)$$

Knowing both optimum induction factors along the rotor blade span, the optimum twist angle, $\theta_{opt}$, can be calculated from the inflow angle which is defined by equation (5) and the optimum chord distribution, $c$, can then be calculated with equation (6). The angle $\alpha_{opt}$ is defined as the angle of attack of the aerofoil with the highest lift-to-drag ratio.

$$\theta_{opt} = \phi - \alpha_{opt} \quad (4)$$

$$\tan \phi = \frac{1 - \frac{a}{\lambda}}{x \left(1 + \frac{a'}{\lambda}\right)} \quad (5)$$

$$c = \frac{4x^2a'}{\sqrt{(1 - \frac{a}{\lambda})^2 + x \left(1 + \frac{a'}{\lambda}\right)^2}} \left(\frac{1 - \frac{a}{\lambda}}{1 - a}\right) \frac{2\pi R}{B\lambda C_{lift}} \quad (6)$$

The value $B$ represents the number or rotor blades, $R$ the radius of the rotor tip, $\lambda$ the tip speed ratio and $C_{lift}$ the lift coefficient at $\alpha_{opt}$. The factor $F$ in equations (5) and (6) is a loss factor modelling the effects of the hub and tip vortices and can be calculated with equation (7)

$$F = \frac{4}{\pi^2} \arccos \left(e^{-f_{tip}}\right) \arccos \left(e^{-f_{root}}\right) \quad (7)$$

where:

$$f_{tip} = \frac{BR - r}{2} \sqrt{1 + \frac{(1 + a')^2x^2}{(1 - a)^2}} \quad (8)$$

$$f_{root} = \frac{BR - R_{root}}{2} \sqrt{1 + \frac{(1 + a')^2x^2}{(1 - a)^2}} \quad (9)$$

This analytical solution considers wake rotation (through $a'$) and takes also hub and tip losses into account using the Prandtl tip loss factor $F$. 


Optimisation process

To determine an optimum rotor blade shape, i.e. chord and twist distribution along the rotor blade span, an optimisation process has been set up. This process consists of a Python script (Van Rossum and Drake, 2009) which uses a differential evolution algorithm in combination with an unsteady BEM solver. With this solver it is possible to calculate the rotor performance in both steady and turbulent wind conditions and use these performance values during the blade optimisation process. All parts of the optimisation process (differential evolution algorithm, BEM solver and Python glue code) are described in more detail in the following three sections.

Differential evolution

The optimum rotor chord and twist distribution is determined by means of a differential evolution (DE) algorithm as described by Storn and Price (1997). In this method a population of randomly distributed individuals is established and the fitness of this population is improved by evolving the fittest individuals. This evolution is accomplished by the exchange of genes from the fittest individuals to a random set of other individuals.

In this case each individual represents a rotor blade with a specific chord and twist distribution (the genes). These distributions are not optimised directly, but the parameters of two BSpline curves (one for the chord and one for the twist distribution) are used as the design variables during the optimisation. This approach has been proposed by Hampsey (2002) to obtain a smooth chord and twist distribution along the rotor blade span.

Both distributions (chord and twist) are represented by a BSpline curve of degree 3 with 5 knots each, resulting in a total of 22 spline coefficients. The number of knots and the degree of 3 has been chosen because they give good results for the steady optimisation as shown in Section “Steady optimum”.

The population size has been chosen to be ten times the number of genes, in this case 220. The initial chord distributions of a population are shown in Figure 1 on the left, with the limits of the spline coefficients set in a fixed range around the analytical optimum to expedite convergence. The population is then evolved over 300 generations and the resulting chord distributions for the whole population are shown in Figure 1 on the right.

![Initial and final population of the chord distribution from the DE optimisation process.](image)

Through the SciPy library (SciPy Community, 2020) an implementation of the differential evolution algorithm is already available and has been used in this study in combination with the Scoop library (Hold-Geoffroy et al., 2014) to distribute the calculation of the fitness of each individual over a cluster infrastructure.

OpenFAST

The objective of the optimisation process is to improve the fitness of each individual which is defined as the mean power coefficient. This objective is determined by an OpenFAST simulation (Jonkman et al., 2020). The OpenFAST software is based on the BEM theory with numerous enhancements, e.g. for the simulation of unsteady aerodynamics or aeroelastic effects.

The SWRT model from the regression test data of OpenFAST has been modified, because the current version (v2.4.0) of OpenFAST is not capable of simulating a wind turbine with a tail fin. Therefore, the yaw degree of freedom has been deactivated (YawDOF = False) and a fixed yaw angle of zero degree has been specified. The influence of this modification is assumed to be low as the prescribed wind distributions have a constant mean wind direction.

In this study, the fitness of the individual is defined by the mean power coefficient of the wind turbine, whereby the mean value has been calculated over the entire simulation time of 610 s with the exception of the first 10 s. Other values
such as starting time, blade root moments, blade noise, annual energy production or others could be included in the future.

In general the BEM theory assumes a steady flow between far upstream and far downstream of the wind turbine, but OpenFAST offers multiple modules to model transient behaviour. First, the dynamic BEM model calculates the unsteady behaviour of the rotor wake and second, a dynamic stall model, based on the Beddoes-Leishman model, calculates the unsteady behaviour of the aerofoil aerodynamics.

For the turbulent simulations multiple wind fields with a duration of 610 s have been generated using the tool TurbSim. TurbSim can generate wind fields with stochastic inflow turbulence “that reflect the proper spatio-temporal turbulent velocity field relationships seen in instabilities associated with nocturnal boundary layer flows” (National Renewable Energy Laboratory, 2020, para. 1). In the settings of TurbSim the turbulence model is set to the IEC Kaimal spectral model which generates a turbulent wind field according to the standard for SWT (IEC 61400-2) using the normal turbulence distribution. During one optimisation process the turbulent wind field is kept constant to ensure the comparability of the fitnesses (mean power coefficients) between the different individuals (blade shapes). The influence of a specific turbulent wind field on the optimum rotor blade shape is investigated later in this paper by modifying the random seed of the turbulence model. Further details about the generation of turbulent wind fields can be found in Jonkman and Buhl (2006).

During the steady simulations, additionally a fixed rotational speed has been specified to obtain a constant tip speed ratio equal to the design value of 6. During the turbulent simulations, the generator degree of freedom has been activated and the rotor speed was free to adapt its rotational speed to the prevailing wind conditions.

Glue code

The glue code for the optimisation process is a Python script that takes the BSpline coefficients from the individuals of the DE optimisation process, and updates chord and twist values at each blade span position (BlSpn) of the AeroDyn blade input file. It also uses the chord distribution to update the blade mass density values (BMassDen) in the ElastoDyn blade input file, as this value influences the inertia of the rotor, which is important for the behaviour of the wind turbine in turbulent wind. To calculate the blade mass density, a constant density value of the whole blade is computed based on the area of the aerofoil, as well as the chord length and weight of the original blade.

RESULTS AND DISCUSSION

Steady optimum

As a means of validation, two optimisations were carried out under steady conditions using the numerical optimisation process described before. The first optimisation neglects losses due to hub and tip vortices and the second considers both. The results in form of the obtained optimum chord and twist distributions are shown in Figure 2. Additionally, the optimum chord and twist distributions according to the analytical approach are plotted. It can be seen that for the calculations without hub and tip losses the agreement between the differential evolution (DE) and the analytical solution is very good; only minor differences are visible for both the chord and twist distributions.

![Figure 2 Optimum chord and twist distribution according to analytical solution and obtained from differential evolution (DE) both without and with hub and tip losses (HL+TL)](image)

Looking at the simulations with hub and tip losses, some differences can be observed between the analytical and DE solution, especially in the hub region of the twist distribution. Both solutions reduce the twist angle at the blade root and tip. For the hub region a good agreement is only visible for the chord distribution, the DE solution does not reduce the twist angle at the blade root as much as the analytical solution. This is due to the low influence of the blade root region to the
power coefficient of the rotor blade. The blade root region can be optimised to improve the starting behaviour of SWTs as Clifton-Smith and Wood (2007) showed.

Based on this good agreement of the steady optimisation result with the analytical solution, it can be stated that the numerical optimisation process has been devised correctly and produces reasonable results.

Turbulent optimum

The numerical optimisation with turbulent wind yields a slightly different optimum rotor blade shape compared to the steady optimisation as shown in Figure 3. The turbulent optimisation yields only in the root region the same chord and twist values as the steady optimum. For larger radial positions the chord and twist distributions differ from the steady optimum. The effect of these differences on the rotor performance will be discussed later in this paper. First, the influence of the simulation duration and the actual turbulent wind field on the rotor blade shape will be investigated.

![Figure 3 Optimum chord and twist distribution gained by an optimisation with steady and turbulent wind](image)

Under steady conditions the duration of the simulation can be kept very short, as the aerodynamics do not change over time. Whereas, under turbulent conditions, it is important to simulate a sufficient amount of time to obtain the rotor performance in the time-varying wind. To assess the influence of the simulation time on the optimisation result the turbulent optimisation has been conducted with three different turbulent wind fields and four different simulation durations. The wind fields differ only in the random seed used to generate the wind distribution, other parameters, for example the mean wind speed at hub height and the turbulence intensity, have been kept constant.

![Figure 4 Optimum chord and twist distribution for different turbulent wind fields (TWF) with different random seeds and different simulation durations between 60 s and 600 s](image)
The results of this investigation are shown in Figure 4 in form of the chord and twist distributions for the different durations and wind fields. It can be seen that for a shorter duration the optimisation results differ for the different turbulent wind fields. For a simulation duration of 600 s the differences almost vanish. Therefore, it can be stated that the results of the turbulent optimisation process become independent of the actual turbulent wind field, if the duration of the simulation is chosen to be at least 600 s. It is assumed that a longer simulation duration leads to a better averaging result as the individual turbulent wind fluctuations are less significant for longer simulation durations.

The effect of the turbulent optimisation on the rotor performance is shown in Figure 5, where the mean power coefficients of the optimised rotor blade shapes are shown for the mean wind speed of 5 m/s and turbulence intensities between 0 % and 30 % in comparison to the original straight rotor blade. It can be seen that both optimisations yield a significantly higher mean power coefficient. The performance of the wind turbine with steady optimised blades decreases constantly with increasing turbulence intensity. With the turbulent optimised blades, the mean power coefficient firstly increases slightly with increasing turbulence intensity, and decreases only for turbulence intensities higher than 15 %. This results in a slightly lower rotor performance of the turbulent optimised blades compared to the steady optimisation at low turbulence intensities up to about 13 % and slightly higher rotor performance at higher turbulence intensities.

It can be seen that the performance of the rotor blade shape from the turbulent optimisation is higher at higher turbulence intensities than from the steady optimisation. This performance gain, however, is accompanied by a loss of performance at lower turbulence intensities.

![Figure 5 Mean power coefficient against turbulence intensity of a wind turbine rotor with blades according to the steady and turbulent optimum shape (chord and twist) as well as the original rotor blade shape (left). Detail of the mean power coefficient plot with only steady and turbulent optimum blade shape (right).](image)

To estimate the effect of the turbulent optimisation on the AEP of the SWRT the power curve has been calculated with the turbulent and steady optimised rotor blade as well as with the original rotor blade. These power curves are shown in Figure 6 on the left. They have been determined by simulating the SWRT at different wind speeds, where the corresponding wind fields have been generated with a turbulence intensity according to the standard for small wind turbines, the IEC 61400-2 from the International Electrotechnical Commission et al. (2013). The SWRT with rotor blades optimised under turbulent wind performs at all wind speeds slightly better than the SWRT with rotor blades from the steady optimisation. The original blade performs best at wind speeds higher than 12.5 m/s.

In order to estimate the AEP, a wind speed distribution at a reference site is necessary. From former measurements a typical wind speed distribution is known in form of a Weibull distribution with shape parameter $k = 1.7795$ and scale parameter $s = 5.7132$ m/s. The mean value of this wind distribution is 5 m/s and this distribution is shown in Figure 6 on the right. The AEP is then approximated with Equation (10).

$$\text{AEP} = \frac{\sum_i P(U_i) + \sum_i P(U_i-1)}{2} \left\{ \exp \left[ - \left( \frac{U_i}{s} \right)^k \right] - \exp \left[ - \left( \frac{U_i-1}{s} \right)^k \right]\right\} \frac{8760 h}{a}$$

(10)

This results for the SWRT with the original blades in an AEP of 20650 kWh, with the blades from steady optimisation in 24450 kWh and with the blades from the turbulent optimisation in 24950 kWh. This represents a gain of approx. 3800 kWh (or 18 %) and approx. 4300 kWh (or 20 %) compared to the original blade for the steady and turbulent optimised rotor blades, respectively. The differences between the AEP of the steady and turbulent optimum are only small.

Finally, the computational effort of both optimisations should be discussed. For the steady optimisation it is necessary to simulate only a very few time steps for each individual, as the aerodynamics do not change with a constant rotor and wind speed. One such simulation took about 0.146 s. During the DE optimisation the fitness of all individuals of the population (220 individuals) had to be simulated at each iteration (300 iterations). This results in a total of 66 000 simulation runs taking
about 2.7 h. As shown previously, for the turbulent optimisation much longer simulation durations are necessary to obtain an optimum rotor blade shape independent of the actual wind field used. The simulation of one individual of the population took in this case about 387.3 s, which resulted in a total simulation time of 7100 h. The optimisation was parallelised to reduce the wall time. The calculation of the fitness of each individual can be distributed to a separate CPU. The maximum parallelisation in this case is over 220 CPUs which results in a wall time of approx. 33 h. Regarding the relatively low gain in the AEP, it is questionable if the huge investment of computational effort of transient simulations is worth in order to improve the rotor blade shape.

**CONCLUSIONS**

A rotor blade optimisation process has been devised to assess the necessity of transient simulations for rotor blade shape optimisation. This process has been validated using an analytical optimisation result, and was then used in combination with turbulent wind fields generated with the NREL tool TurbSim to investigate the influence of high turbulence intensities on the optimum rotor blade shape.

It could be shown that the optimum rotor blade shape using turbulent wind differs from the steady optimum. Higher chord and twist values are obtained, especially at larger radial positions along the rotor blade span. This new optimum rotor blade shape results in higher mean power coefficients of the SWRT rotor at turbulence intensities above 13 % and a gain in the AEP for a reference site of about 2 % compared to an analytically optimised rotor blade shape.

The computational effort necessary to obtain an optimum rotor blade shape is much higher for turbulent wind conditions than for steady conditions. The use of a computational cluster infrastructure is necessary to obtain the results within a reasonable time frame. It is questionable if this additional computational effort is worth the small gain in the AEP.

**NOMENCLATURE**

- $\alpha_{opt}$: Optimum angle of attack, with highest lift-to-drag ratio
- $\lambda$: Tip speed ratio
- $\omega$: Rotational speed
- $\phi$: Local inflow angle, Eq. (5)
- $\theta_{opt}$: Optimum twist angle, Eq. (4)
- $a$: Axial induction factor, Eq. (1)
- $a'$: Tangential induction factor, Eq. (3)
- $B$: Number of rotor blades
- $c$: Blade chord length, Eq. (6)
- $C_{drag}$: Drag coefficient
- $C_{lift}$: Lift coefficient
- $F$: Loss factor, Eq. (7)
- $k$: Shape parameter
- $R$: Rotor tip radius
- $r$: Radial position
- $R_{root}$: Rotor root radius
- $s$: Scale parameter
- $V_0$: Wind speed
- $x$: Local tip speed ratio, Eq. (2)
REFERENCES


