MULTI TRAVELING WAVE FLUTTER ONSET IN TUNED BLADED-DISKS

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ABSTRACT

Flutter is a major constraint on modern turbomachines; as the designs move toward more slender, thinner, and loaded blades, they become more prone to experience high cycle fatigue problems. Dry friction, present at the root attachment for cantilever configurations, is one of the main sources of energy dissipation. It saturates the flutter instability growth producing a limit cycle oscillation whose amplitude depends on the balance between the energy injected and dissipated by the system. Since both phenomena, flutter and friction, are small corrections of the elastic behavior of the structure, it takes many elastic oscillations for them to manifest and modulate the oscillation amplitude. Furthermore, even longer time scales appear when multiple traveling waves are aerodynamically unstable and exhibit similar growth rates, with a very slow energy transfer rate between them. All these slow scales make the system time integration very stiff and CPU expensive, bringing some doubts about whether the final solutions are properly converged. In order to avoid these uncertainties, a numerical continuation procedure is applied to analyze the solutions that set in, their traveling wave content, their bifurcations and their stability. The system is modeled using an asymptotic ROM and the continuation results are validated against direct time integrations. New final states with multiple traveling wave content are found and analyzed. These solutions have not been obtained before for the case of microslip friction at the blade attachment; only solutions consisting of a single traveling wave have been reported in previous works.

INTRODUCTION

Flutter onset is currently a very important limitation in modern turbomachinery, where blade designs are getting more slender and closer to their mechanical limit. It is an aeroelastic instability where the gas flowing around the blades tends to amplify the small elastic oscillations of the blades. As a consequence, the blade vibration amplitude grows exponentially until nonlinear effects, typically due to friction forces at the blade root in the case of Low Pressure Turbines (LPT), become relevant. Hopefully, dissipation produced by nonlinear friction is strong enough to limit the growth of the aeroelastic instability and avoid a catastrophic blade failure. In this case, there is a balance between nonlinear friction damping and unstable flutter growth such that a final Limit Cycle Oscillation (LCO) state sets in with a limited blade vibration amplitude.

The determination of this final vibration amplitude is crucial to estimate the blade fatigue level. This typically requires a CFD description of the aerodynamic flow coupled with a FEM for the motion of the bladed-disk with nonlinear friction effects. These simulations can include a number of simplifications (linearized CFD, partial fluid-structure coupling, reduced number of modes for the motion of the structure,...), but they are always quite involved and CPU costly.

In a tuned configuration, if the final state is made of a single Traveling Wave (TW), then the calculations can be reduced to just one flow passage and one sector of the bladed-disk. This constitutes a huge computational reduction in the usual case of large blade counts.

For this reason, there has recently been some research activity looking into the expected TW composition of the final LCO state. Aeroelastic models of the bladed-disk with different levels of detail have been used to try to decide whether, in a tuned rotor, the final LCO state is always made of a single TW or if there can also be multi-TW final states.

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Previous works on flutter instability including microslip friction (Corral and Gallardo, 2014; Martel et al., 2014; Martel and Corral, 2013) have always reported single TWs as final states. This TW was the most aeroelastically unstable or one of the adjacent ones. On the other hand, in the case of strong nonlinear coupling effects (contact at the tip shroud), multi-TW states have been reported both in reduced models (Krack et al., 2017) and more realistic descriptions (Gross and Krack, 2019). Further analysis (Berthold et al., 2020) showed that nonlinear tip contact effects change the vibration modes and frequencies of the system with respect to those obtained by linearized contact conditions.

In the present work, we consider the case of flutter vibrations with microslip contact forces at the blade fir-tree, where the effect of friction does not substantially change the natural vibration modes. As it was mentioned above, previous work on this configuration (Corral and Gallardo, 2014; Martel et al., 2014; Martel and Corral, 2013) have only found final vibration states consisting of just one single TW. The idea is to apply numerical continuation techniques (Doedel et al., 2007) to analyze the stability of the solutions and search for multi-TW states. This procedure eliminates the need for direct time integration of the system, which requires very long integration times, and always leaves some uncertainty about whether the final state is fully converged.

To this end, first, we briefly introduce the asymptotic model used, study the stability of the zero solution (no vibration) and compute the single TW solutions that appear when the magnitude of the flutter instability is increased. Then, we analyze the linear stability of single TW solutions and perform a numerical continuation to follow new branches of multi-TW solutions that bifurcate from single TW solutions when they lose stability. New multi-TW states are obtained. They are quasi-periodic in time with nonuniform blade to blade amplitude and propagate along the rotor.

ASYMPTOTIC MODEL

The derivation of the asymptotic model is completely analogous to that explained in detail in (Martel et al., 2014; Martel and Corral, 2013), and is omitted here. This model is expressed in terms of complex TW amplitudes $A_j$ can be written in the form

$$
\frac{d}{d\tau}\begin{pmatrix}
A_1 \\
\vdots \\
A_N
\end{pmatrix} = \left( [M_{\text{friction}}] + [M_{\text{aero}}] + [M_{\text{elastic}}] \right) \begin{pmatrix}
A_1 \\
\vdots \\
A_N
\end{pmatrix},
$$

(1)

where $N = 24$ is the number of blades of the rotor. The elastic and aerodynamic matrices are diagonal in the TW basis

$$
[M_{\text{elastic}}] = \begin{pmatrix}
-\frac{\xi_{\text{mat}}}{\theta} + i\frac{\Delta \omega_1}{\theta} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & -\frac{\xi_{\text{mat}}}{\theta} + i\frac{\Delta \omega_N}{\theta}
\end{pmatrix},
[M_{\text{aero}}] = \begin{pmatrix}
-\frac{\xi_1}{\theta} + i\frac{\eta_1}{\theta} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & -\frac{\xi_N}{\theta} + i\frac{\eta_N}{\theta}
\end{pmatrix},
$$

(2)

and the friction matrix is diagonal in the displacement basis

$$
[M_{\text{friction}}] = -\frac{i}{2} [E]^H \begin{pmatrix}
Q(|X_1|) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & Q(|X_N|)
\end{pmatrix} [E],
$$

(3)

where $[E]$ is the change matrix from TW amplitudes to blade displacements and $[E]^H$ denotes the conjugate transpose of $[E]$. The complex blade displacements are represented by $X_j$ and they are related to $A_j$ by $X = [E]A$.

![Figure 1 Elastic Frequencies of the Bladed-Disk.](image)

The coefficients $\Delta \omega_j$ in the elastic matrix represent the small deviations of the natural frequencies of the modal family from the blade alone frequency $\omega_0 = 1$ (see Fig. 1). The material damping coefficient of the structure has the value $\xi_{\text{mat}} =$
The complex friction coefficient $Q(|X_j|)$ in the friction matrix $[M_{\text{friction}}]$ accounts for the nonlinear dissipation and frequency change produced by friction at the fir-tree (see Martel et al., 2014; Martel and Corral, 2013), and depends on the amplitude of the blade elastic cycle $|X_j|$. Real and imaginary parts of $Q(|X_j|)$ are shown in Fig. 2 for the Olofsson microslip friction model (Olofsson, 1995).

![Figure 2 Real (top) and Imaginary (bottom) Parts of the Complex Friction Coefficient](image)

The coefficients in $[M_{\text{aero}}]$ are the aerodynamic damping and frequency correction of the TW modes. A sinusoidal shape (blade interaction with itself and with its adjacent blades only) has been selected to mimic the typical data in a realistic Low Pressure Turbine (see, e.g., Corra et al., 2018)

$$\xi^j_a = \xi_{a0} - \xi_{a1} \sin \left( \frac{2\pi j}{N} + \xi_{a2} \right) \%,$$

$$\eta^j_a = \eta_{a0} - \eta_{a1} \cos \left( \frac{2\pi j}{N} \right) \%,$$

where $j$ is the TW wavenumber, $\xi_{a0} = 0.25$, $\xi_{a1} = 0.75$, $\xi_{a2} = -0.05$, $\eta_{a0} = 1.4$ and $\eta_{a1} = 1$. The aerodynamic coefficients are shown in Fig. 3. The aerodynamically unstable TW modes (i.e., $\xi^j_a < 0$) are represented with a shaded area. Note that the mean aerodynamic damping is stable and there are 9 unstable TWs.

The coefficient $\xi_{a1}$ will be used later on as the bifurcation parameter to explore the solutions of the system for different levels of flutter instability.

**TRAVELING WAVE SOLUTIONS**

Eq. (1) admits the trivial solution $A_j = 0$ for $j = 1, \ldots, N$, which corresponds to no blade motion. Linearizing the problem around this solution yields

$$\frac{d}{d\tau} \begin{pmatrix} \vdots \\ a_j \\ \vdots \end{pmatrix} = \text{diag} \left[ -\xi^j_a + \frac{\xi_{\text{mat}}}{\theta} i \left( \Delta \omega_j + \frac{\eta_j}{\theta} - \Re[Q(0)] \right) \right] \begin{pmatrix} \vdots \\ a_j \\ \vdots \end{pmatrix} ,$$

where $a_j$ represents the perturbed TW amplitude.

This is a constant coefficient system and the eigenvalues of the matrix are given by

$$\lambda_j = -\xi^j_a + \frac{\xi_{\text{mat}}}{\theta} i \left( \Delta \omega_j + \frac{\eta_j}{\theta} - \Re[Q(0)] \right) .$$
As flutter intensity is increased through the parameter $\xi_{a1}$ (see eq. (4)), the trivial solution becomes unstable when $\frac{\xi_{a1} + \xi_{mat}}{\theta} < 0$ for any $j$. As it can be seen in Fig. 3, this happens first for $j = \text{ND} = 6$, which is the most aeroelastically unstable TW mode. For higher values of $\xi_{a1}$, the adjacent TW modes become also unstable. The effect of friction is just a shift in frequency of magnitude $\Re[Q(0)]/2$ in the aeroelastic modes.

There is another type of simple solutions of the system, which corresponds to a single TW with all blades vibrating with the same amplitude and frequency, and with a given wavenumber $r$

$$A_r = \sqrt{N} R_r e^{i(m_r \tau + \alpha)}, \quad A_j = 0 \text{ for } j = 1, \ldots, N \text{ and } j \neq r,$$

where $\sqrt{N} R_r$ is the modulus of the TW amplitude, $m_r$ is the frequency correction and $\alpha$ represents a free phase. The values of $R_r$ and $m_r$ are given by

$$\Im[Q(R_r)] = \frac{2(\xi_{a1} + \xi_{mat})}{\theta}, \quad (9)$$

$$m_r = \frac{\Delta \omega_r + \eta_r}{\theta} - \frac{1}{2} \Re[Q(R_r)]. \quad (10)$$

Eq. (9) indicates that the amplitude of the TW is the result of the balance between the growth rate of the aeroelastic instability and the nonlinear damping of the friction (Martel et al., 2014). After computing the amplitude $R_r$, the frequency correction $m_r$ can be directly obtained from eq. (10).

For each unstable TW mode with wavenumber $j = r$ there is a nonlinear TW with amplitude $A_r$ that bifurcates from the trivial solution precisely when the real part of the eigenvalue $\lambda_r$ from eq. (7) becomes positive. The different nonlinear TWs bifurcating from the zero solution as the flutter intensity parameter $\xi_{a1}$ is increased are represented in Fig. 5. The trivial solution becomes unstable when $\xi_{a1}$ crosses the straight dotted line which corresponds to the first TW mode becoming unstable (ND = 6).

**STABILITY OF THE TW SOLUTIONS**

The stability characteristics of the TW solutions are now obtained by linearizing the system of eqs. (1). To this end, the solution is written as a single TW $A_r$ (with wavenumber $r$, see eq. (8)) plus a small perturbation
\[
\begin{pmatrix}
A_1 \\
\vdots \\
A_r \\
\vdots \\
A_N
\end{pmatrix} = \begin{pmatrix}
0 \\
\vdots \\
1 \\
\vdots \\
0
\end{pmatrix} + \begin{pmatrix}
a_1 \\
\vdots \\
a_r \\
\vdots \\
a_N
\end{pmatrix} \sqrt{N} R e^{i(m_r \tau + \alpha)}.
\]
(11)

Expression (11) is introduced in the system of eqs. (1) and only linear terms in the perturbation are retained. After some manipulations, the resulting linear system for the time evolution of the perturbation can be expressed as

\[
\frac{d}{d\tau} \begin{pmatrix}
a_{r-1} \\
a_r \\
\vdots \\
a_{r+1}
\end{pmatrix} = -i \frac{4}{Q'(R)} R_e \begin{pmatrix}
\alpha_{r-1} \\
\alpha_r \\
\vdots \\
\alpha_{r+1}
\end{pmatrix} + \begin{pmatrix}
\bar{a}_{r+1} \\
\bar{a}_r \\
\vdots \\
\bar{a}_{r-1}
\end{pmatrix} + \text{diag} \left[ -\frac{\xi_0^j - \xi_0^r}{2\eta} + i \left( \frac{\Delta \omega_j - \Delta \omega_r + \eta_j^k - \eta_r^k}{\theta} \right) \right] \begin{pmatrix}
a_{r-1} \\
a_r \\
\vdots \\
a_{r+1}
\end{pmatrix}.
\]
(12)

Note that the second term in system (12) contains the aerodynamic damping differences \(\xi_j^a - \xi_r^a\). Given that, close to the most aeroelastically unstable TW, the aerodynamic dampings are very similar, these differences are small and generate even longer time scales. In Fig. 4, the results of a time integration starting from a small initial condition \(A_j = 0.01\) for every \(j\) and \(\xi_a^1 = 0.75\) are shown. Initially, there is an exponential growth of aeroelastically unstable TW modes (straight lines due to logarithmic scale). The slope of this growth (blue dashed line) is practically given by the most unstable aerodynamic damping \(\xi_6^a\) (the material damping of the structure is much smaller). However, the slope of the TW decay in the nonlinear interaction region (red dashed line), which is related to the difference of aerodynamic dampings \(\xi_j^a - \xi_r^a\), is much smaller and it takes a very long time to select the final TW with wavenumber 6. Direct time integration, even for this asymptotic model, requires long computation times, and there is always a certain level of doubt about whether the convergence to the final state has been reached.

![Figure 4 Time Evolution of the Solution of System (1).](image)

The stability of different single TW solutions has been obtained by calculating the eigenvalues of the matrix in system (12) and checking the signs of its real parts. The results are plotted in Fig. 5 as a function of the bifurcation parameter \(\xi_{01}\), which controls the intensity of the flutter instability. The TW solution with wavenumber 6 (which corresponds to the most unstable aerodynamic damping) is always stable, but adjacent ones (with slightly less unstable aerodynamic damping) only become stable as
the bifurcation parameter $\xi_{aj}$ is increased. This possibility of having different single TW solutions as final states was also found in (Martel et al., 2014) through the direct time integration of system (1).

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodal Diameter</td>
<td></td>
<td></td>
<td></td>
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</table>

**Figure 5** Bifurcation Diagram of the Traveling Wave Solutions From The Trivial Solution. Solid (Dashed) Line Indicates Stable (Unstable) Solutions.

**MULTI-TW SOLUTIONS**

We now look for more complicated solutions that can bifurcate from single TW solutions. The numerical continuation software AUTO (Doedel et al., 2007) is used to track these new branches, but first the following change of variables is made to system (1)

\[
\begin{pmatrix}
A_j
\end{pmatrix}
= \begin{pmatrix}
B_j
\end{pmatrix} e^{im_\tau},
\]

(13)

to remove the frequency of the TW solutions and turn them into steady solutions. After making the change of variable, the equations of motion take the form

\[
\frac{d}{d\tau}
\begin{pmatrix}
B_1
\vdots
B_r
\vdots
B_N
\end{pmatrix}
= -\frac{i}{2} [E]^H \text{diag} [Q(|X_j|) - Q(|R_j|)] [E] \begin{pmatrix}
B_1
\vdots
B_r
\vdots
B_N
\end{pmatrix}
+ \text{diag} \left[ \frac{\xi_j - \xi_{ra}}{\theta} + i \left( \frac{\Delta \omega_j - \Delta \omega_a + \eta_{ja} - \eta_{ra}}{\theta} \right) \right] \begin{pmatrix}
B_1
\vdots
B_r
\vdots
B_N
\end{pmatrix},
\]

(14)

where the differences of aerodynamic dampings $\xi_j - \xi_{ra}$ are present in the linear part of the system.

The results show that a Hopf bifurcation (HB) occurs at the point where there is a stability change in the single TW solutions branch. The solutions that come out of this HB are periodic in time. In the original system (1), these solutions are actually quasi-periodic, since we have removed the TW frequency with the change in eq. (13). Almost every new solution found is unstable, except for the branch corresponding to the TW with wavenumber 4, where there is a small region in the bifurcation parameter with stable multi-TW solutions.

The new solutions are represented in Fig. 6. The multi-TW solution branch appears through a backwards HB from the single TW solution with wavenumber 4. This multi-TW solution is stable only for very narrow range of values of the bifurcation parameter, but its TW components grow to a considerable amplitude (see the inset in Fig. 6). As one continues decreasing the bifurcation parameter, a Torus bifurcation occurs and the multi-TW solution loses stability.
Finally, the displacements of the blades $|X_j|$ of the new multi-TW solution for $\xi_{a1} = 0.502$ are plotted in Fig. 7. The multi-TW solution takes the form of a traveling wave with non-uniform amplitude that rotates around the bladed-disk. Despite the complexity of the multi-TW solutions that are obtained in this work, the maximum blade displacement that they produce is always smaller than that associated to the single TW solution with highest amplitude (corresponding to the most aerodynamically unstable wavenumber 6). As presented in Table 1, the maximum blade displacement of the TW solution is 17% higher than that of the multi-TW solution. Therefore, from the point of view of the calculation of the fatigue of the blades, it seems that it is conservative to consider only the highest amplitude TW solution.

<table>
<thead>
<tr>
<th>Multi-TW</th>
<th>TW6</th>
<th>Relative Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.224</td>
<td>0.269</td>
<td>17 %</td>
</tr>
</tbody>
</table>

Table 1 Comparison of Maximum Blade Displacements
CONCLUSIONS

Multi-TW stable states are found in a simplified model of a bladed-disk with microslip friction at the bladed-disk attachment. The model describes the vibration of a nearly flat modal family with several aeroelastically unstable modes, and is derived through the application of a multiple scales method that allows to filter out the fast elastic blade oscillation. Numerical continuation techniques are applied to follow the branches of new multi-TW solutions that bifurcate from single TW solutions at the stability change. Previous works reported only single TW final states, which were obtained through direct time integrations of the model. The continuation method directly computes the final states and their stability, avoiding the need of performing very long time integrations to reach a converged final state.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>HCF</td>
<td>High Cycle Fatigue</td>
</tr>
<tr>
<td>HB</td>
<td>Hopf Bifurcation</td>
</tr>
<tr>
<td>LCO</td>
<td>Limit Cycle Oscillation</td>
</tr>
<tr>
<td>LPT</td>
<td>Low Pressure Turbine</td>
</tr>
<tr>
<td>ND</td>
<td>Nodal Diameter</td>
</tr>
<tr>
<td>ROM</td>
<td>Reduced Order Model</td>
</tr>
<tr>
<td>TW</td>
<td>Traveling Wave</td>
</tr>
<tr>
<td>$A_j$</td>
<td>Complex TW amplitude</td>
</tr>
<tr>
<td>$Q(X)$</td>
<td>Complex friction coefficient</td>
</tr>
<tr>
<td>$X_j$</td>
<td>Complex blade displacement</td>
</tr>
<tr>
<td>$\eta^a_j$</td>
<td>Aerodynamic frequency correction</td>
</tr>
<tr>
<td>$\xi^a_j$</td>
<td>Aerodynamic damping</td>
</tr>
<tr>
<td>$\xi_{mat}$</td>
<td>Viscous damping coefficient of the structure</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Slow time scale</td>
</tr>
<tr>
<td>$\Delta \omega_j$</td>
<td>Natural frequency deviation</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of blades</td>
</tr>
<tr>
<td>$m_r$</td>
<td>Frequency correction of a single TW</td>
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<tr>
<td>$\alpha$</td>
<td>Free phase of a single TW</td>
</tr>
<tr>
<td>$\xi_{bwj}$</td>
<td>Bifurcation parameter</td>
</tr>
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