Research on Aerodynamics and Aeroacoustics of Duct Fan based on Panel-Vortex Particle Method

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Abstract
With the urge requirement of high-performance aircraft electric propulsor, duct fan is expected to work in different operation conditions efficiently and satisfying multiple targets constraint. However, the putting up of higher design standards means FEM or FVM will be unsuitable because of the high computational cost. To achieve a balance between computational cost and accuracy, especially an unsteady aerodynamic problem e.g. aeroacoustics/aeroelasticity, the high-efficiency Panel-Vortex Particle Method (PVM) is developed to predict the aerodynamic performance of a duct fan. Then, the duct acoustic model is developed to complete the acoustics prediction of duct fan. The hybrid method, consisting PVM and duct acoustic model, can be used to get a similar thrust with error of 0.93% when the span ratio of mesh is less than 91.6%, when comparing with FVM results. But the thrust of blade tip is over-predicted, which causes a total error of 5.36%, and the SPL is in a relative error of 4.16% under 1BPF and 11.55% under 2BPF.

Keywords:
panel method, vortex particle method, duct fan, unsteady aerodynamic, acoustic noise

1 Introduction
With the increasing number of aerospace vehicles around the world, the requirements for high efficiency, energy saving and low noise are becoming higher and higher (Davies et al., 2013). The electric propulsion system is able to use more than 70% of the total electricity to produce thrust (Huang et al., 2016), which meets the needs of energy saving and noise reduction. However, the new concept of distributed electric propulsion aerodynamic layout brings new challenges to the simulation. For example, there are multiple duct fan propulsion units in the aircraft, and when to predict the noise performance of duct fan, it is necessary to carry out the unsteady numerical calculation with great computation cost (Wang et al., 2020). Therefore, it is necessary to develop an efficient numerical simulation method to calculate and analyze the unsteady aerodynamic characteristics of the small duct fans, and the theoretical guidance for advanced power layout design will be provided in this method.

The panel method uses the Green function to convert the Laplace equation of the potential function into a spatial potential flow field with the influence of a solid wall (Hess et al., 1964). It is currently the most accurate potential flow method (Kinnas, 1994), so it is widely used in turbo machinery field (Baltazar et al., 2012) used this method to calculate the performance of duct propellers, and found that the calculation accuracy of the wake vortex is extremely important to the accuracy of performance prediction.

Although the panel method has been widely used in numerical simulation of turbo-machinery, the non-thickness wake assumption will affect the accuracy of numerical calculation when dealing with lift body, so it is necessary to use vortex particle which has thickness. (Wang et al., 2017) simulated the propeller wake using the vortex particle method and analyzed the interference between the rotating vortex particle wake and the wing. (Willis et al., 2010) used a panel-vortex particle hybrid method to solve the wing vibration problem and obtained accurate results. (Tan et al., 2013) used the panel-vortex particle hybrid method to simulate the unsteady aerodynamic characteristics of helicopter during hovering and forward flight conditions, and observed obvious tip and root vortex. (Hao et al., 2015) used hybrid method to calculate the aerodynamic characteristics of wind turbines, and compared the pressure distribution, thrust and torque at different blade span percentages with the experimental results. (Chatelain et al., 2018) used large quantity of vortex...
particles to simulate the wingtip vortex and finds that this method can accurately simulate the tip vortex structure. However, the above-mentioned researches mainly use the hybrid method to carry out numerical simulation for wing or external-flow turbo machinery, and the research on internal-flow duct fan is still limited.

The aerodynamic and acoustic simulation of turbomachinery can be realized by combining the aerodynamic method introduced by the above method with the acoustic radiation calculation method. Acoustic analogy method is widely used in sound propagation calculation, and corresponding models are developed for free sound field and pipe sound field.

In order to apply its acoustic analogy method to the noise prediction of rotating turbomachinery, (J. E. Fowcs Williams et al, 1969) extended Lighthill equation to the flow sound problem with arbitrary moving solid boundary. (Goldstein, 1976) derived the solution of sound radiation in an infinite circular duct, and the non-reflecting pipe wall boundary and uniform incoming flow when deriving the pipe acoustic model, which is more in line with the actual situation.

In this paper, the panel-vortex particle hybrid method coupled with the duct acoustic model method are combined to realize the fast prediction of the aerodynamic and acoustic performance of the duct fan. The feasibility of using the hybrid method PVM to predict the aerodynamic and noise performance is verified. At the same time, the mathematical treatment methods to improve the accuracy of panel and vortex particle calculation are also discussed.

2 Methodology

2.1 Panel method

Based on the continuity of fluid and the concept of potential function, the panel method uses Green's function to solve Laplace equation in three-dimensional space to obtain velocity potential. Source, sink and dipole are introduced as potential flow units. For P and Q on the solid wall, there are two points:

\[ \nabla^2 \phi = 0 \]  

\[ -4\pi \varepsilon_0 E(\rho) = \oint S (\frac{\partial}{\partial n} Q \frac{1}{R_{pq}} - \frac{1}{R_{pq}} \phi \frac{\partial \phi}{\partial n}) dS \]  

With boundary conditions:

\[ \left\{ \begin{array}{l}
\nabla \phi = 0 & \text{on } S_b \\
\n\nabla \phi \rightarrow 0 & \text{on } S_w \rightarrow \infty \\
\n\frac{\partial \phi}{\partial n} = 0 & \text{on } S_w
\end{array} \right. \]

Therefore, the velocity potential can be represented by using the dipole and the source, obtained from Formula 2:

\[ 4\pi \varepsilon_0 E(\rho) = \oint S (\frac{\partial Q}{\partial n} \frac{1}{R_{pq}} + \left[ \frac{\partial}{\partial n} (\frac{1}{R_{pq}}) \phi \right] \frac{1}{R_{pq}} dS + \oint S \nabla \phi Q w \left( \frac{1}{R_{pq}} \right) dS 
\]

\[ \Delta \phi (Q_w) = \phi (Q_w) - \phi (Q_{in}) \]

In order to avoid the numerical singularity caused by the close distance, the equivalent vortex section is used to calculate the induced velocity of the panel. Therefore, the radius r of the vortex ring is introduced based on Biot-Savart law to eliminate the singularity of numerical calculation.

**Figure 1 Panel and the equivalent vortex filament**

Where R1 is the space vector between the beginning and the end of the vortex section, R2 is the space vector between the end of the vortex section and the field point, R0 is the space vector between the beginning and end of the vortex section, and h is the vertical distance between the field point and the vortex section.

**Table 1 geometry parameters of duct fan**

<table>
<thead>
<tr>
<th>parameter</th>
<th>data</th>
<th>parameter</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>blade number</td>
<td>9</td>
<td>axial chord</td>
<td>40 mm</td>
</tr>
<tr>
<td>inner diameter radius</td>
<td>66 mm</td>
<td>maximum thickness of</td>
<td>3.5 mm</td>
</tr>
</tbody>
</table>
In order to satisfy the Kutta condition of panel method, the blade trailing edge should be treated as a sharp trailing edge. The influence of duct panel on the blade is mainly related to the accuracy of calculating the induced velocity. In this paper, the duct and hub panel with different inclination angles are obtained by adjusting the circumferential dispersion angle and axial dispersion length of the panel. The specific idea is to obtain different inclination angles by controlling the circumferential angle and axial length on the foundation of the hub (casing) radius $R$.

![Image](a) panel discretization method, (b) blade tip, (c) blade root, (d) duct fan

**Figure 2 Panel discretization method of the duct fan**

In 2018, (Kim et al., 2018) explored the relationship between the casing and blade panel matching using different discretization method, which confirmed that the panel size between the blade and the casing has a great impact on the numerical calculation accuracy. So, the casing discretization angle is set as 1.2° and the hub dispersion angle is 1.5° to achieve a good matching.

### 2.2 Vortex particle method

Vortex particle method is a grid free vortex method. Based on the incompressible Navier-Stokes equation, the vorticity transport equation can be obtained by taking the divergence of velocity:

$$\frac{D\omega}{Dt} = \omega \cdot \nabla \mathbf{u} + \nabla \times \mathbf{v} \times \omega$$

(6)

The vorticity in the space is discretized by vortex particles, and the intensity and position of the $i$-th vortex particle are respectively represented as $\alpha_i$ and $\mathbf{x}_i$, then the vorticity field can be discretized as:

$$\omega(x,t) = \sum_{i=1}^{N_p} \xi(x-x_i)\alpha_i = \sum_{j=1}^{N_p} \xi_j(x-x_i)\alpha_j$$

(7)

The smooth core function $\xi_j$ of vortex particles makes the distribution of vorticity smooth. At present, the Gauss form core function is mainly used (Barba, 2004). The vortex particle method uses vortex particles to simulate the development of the flow field, and the strength of the particles changes due to vortex stretching and viscous dissipation. Therefore, the solution process of the vortex particle method is divided into two iterations between position change and vortex intensity change:

$$\frac{dx_j}{dt} = \sum_{i=1}^{N_p} K(\rho) (x_j-x_i)\times \alpha_i (t)$$

(8)

$$\frac{d\alpha_j}{dt} = \alpha_j \sum_{i=1}^{N_p} \nabla K(\rho)(x_j-x_i)\times \alpha_i (t) + \frac{2\nu}{\sigma^2} \sum_{i=1}^{N_p} (V_i/\alpha_j - V_j/\alpha_j)\xi_j(x_j-x_i)$$

(9)

In this paper, the fourth-order Runge-Kutta algorithm is used to solve formula 8 and formula 9. $\rho = |\mathbf{x} - \mathbf{x}_j|/\sigma$ is a dimensionless length parameter, and $K(\rho)$ is a Biot-Savart law kernel function for calculating the induced velocity, and has:
\[ G(\rho) = \frac{1}{4\pi \rho} \text{erf} \left( \frac{\rho}{\sqrt{2}} \right) \]  

(10)

One of the convergence conditions of the vorticity transport equation is the overlap condition (J. Thomas et al., 1986). When the radii of adjacent vortices are crossed, the calculation can be carried out smoothly. Now the constant radius of vortices and the variable radius of vortices are both used in different conditions. Here, the constant radius of vortices is used, and the overlap coefficient is 1.2 to ensure the accuracy of the calculation.

The smooth kernel function can directly point out the distribution of vorticity in space. Therefore, the use of the kernel function can greatly affect the numerical calculation accuracy. The following characteristics are found for the smooth kernel function with r-order precision (Ploumhans et al., 2002):

<table>
<thead>
<tr>
<th>Fun</th>
<th>(4\pi \xi(R))</th>
<th>(4\pi G(R))</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fun1</td>
<td>(\frac{3}{(R^2 + 1)^{3/2}})</td>
<td>(\frac{1}{(R^2 + 1)^{1/2}})</td>
<td>0</td>
</tr>
<tr>
<td>Fun2</td>
<td>(\frac{2}{\pi} e^{-R^2/2})</td>
<td>(\frac{1}{R} \text{erf} \left( \frac{R}{2\sqrt{2}} \right))</td>
<td>2</td>
</tr>
<tr>
<td>Fun3</td>
<td>(\frac{2}{\pi} \left(\frac{5}{2} - \frac{R^2}{2}\right) e^{-R^2/2})</td>
<td>(\frac{1}{R} \left( \text{erf} \left( \frac{R}{2\sqrt{2}} \right) + \frac{R}{2\sqrt{2}} e^{-R^2/2} \right))</td>
<td>4</td>
</tr>
</tbody>
</table>

As mentioned above, the Gaussian distribution function with second-order accuracy is used to ensure the accuracy of numerical calculation, which can realize vorticity concentration and reduce the calculation cost.

### 2.3 Goldstein's duct acoustic model

In the 1970s or so, Goldstein derived duct acoustic modal model based on FW-H equation and non-reflective infinite duct effect, which can be used to predict the aerodynamic noise of fan, compressor and other internal flow turbo-machinery (Goldstein, 1976). In the case of uniform inflow, the Green's function of the wave equation can be obtained as follows:

\[ G_2(\vec{x},\vec{y}) = \frac{1}{2\pi} \sum \psi_n(\vec{y}) \nabla \cdot G_1(\vec{x},\vec{y}) \exp \left\{ \frac{\mu R_1(x_1-x_2)+k_n |x_1-y_1|}{\beta^2} \right\} \]  

(11)

The Green's function of the acoustic dipole can be obtained by differentiating the Green's function, and the differential form of the acoustic dipole can be obtained from the second term on the right of Lighthill's acoustic analogy equation.

\[ p(\vec{x},\omega) = f_1(\tau) \nabla \cdot G_2(\vec{x}) \]  

(12)

At the same time:

\[ \nabla \cdot G_2 = \frac{\partial G_2}{\partial \tau} + \frac{1}{r} \frac{\partial G_2}{\partial r} + f_1 \vec{r} \]  

(13)

The pressure load on the blade surface is vertical to the surface of the blade. On the grid of surface, \( P \) is the pressure, \( S \) is the grid area, and \( \vec{n} \) is the normal vector outside the blade surface. The dipole sound pressure formula in the time domain can be obtained:

\[ p(x,\tau) = \int_0^T \int I(x,\tau) \nabla \cdot G(\vec{x},\vec{y}) \nabla \cdot p(\vec{y},\tau) d\tau d\vec{y} \]  

(14)

In order to analyze the periodic noise of turbo-machinery, the above formula is transformed into frequency domain:

\[ p(x,\omega) = \sum_m \sum_n P_{mn}(\omega) \Psi_{mn}(\vec{x}) \exp(\text{i}m\phi - \text{i}m\omega,\vec{x}) \]  

(15)

The amplitudes of the \((m, n)\) th modes are as follows:

\[ p_{mn}(\omega) = \frac{1}{2\pi \omega} \int_0^{2\pi} \int_0^{2\pi} \Psi_{mn}(\vec{x}) \cdot \vec{n} \cdot \nabla \exp(-\text{i}m\phi + \text{i}m\omega,\vec{x}) d\vec{y} \]  

(16)

### 2.4 Coupling of panel method, vortex particle method and duct acoustics model

In order to obtain more accurate wave, vortex particles that can freely simulate spatial vorticity distribution are used instead of wake panels that can not change their shape freely (Tan et al., 2013):

\[ \nabla \cdot \int \rho \frac{\partial G}{\partial t} dS = - \nabla \cdot \int \vec{G} \cdot dS - \nabla \cdot \int \vec{G} \cdot d\vec{S} \]  

(17)

After the four edges of the wake panel are transformed into vortex filaments, the vortex filaments of the adjacent edges are merged, and the vortex filaments are equivalent to vortex particles for the further solution. In order to meet the Kutta condition of the trailing edge, a column of buffer wake panel is added as the transition between the panel method and the vortex particle method, and the edge of the vortex particles near the trailing edge obtained from the first column is equivalent to the vortex line (red line in Figure 3). The other three edges are transformed into vortex particles, as shown
3 Numerical Results

The noise characteristics of duct fan at 7700rpm are analyzed and the blade passing frequency is 1155Hz, and the size of the blade surface mesh is controlled in order to meet the acoustic requirements. When using URANS and acoustic analogy theory to analyze aerodynamic noise of turbo-machinery, 3BPF is generally chose as the upper limit of analysis. In order to ensure Nyquist sampling principle is satisfied, it is necessary to control sampling frequency no more than 6BPF (here is 6930hz), and the corresponding time step is 3.608×10⁻⁵s, which relates to 24BPF. The sound wavelength of 3BPF is 0.098m, and the grid scale should be less than 0.016m.

The numerical calculation of duct fan using FVM is based on the k-ω SST model, which is usually used in turbo-machinery simulation. The inlet boundary condition is set as velocity of 30m/s, and the temperature is ensured to be 288K. The pressure on the outlet boundary condition is 95100Pa. Internal interface is used as the rotor-stator interface.

In this way, the numerical calculation condition is consistent with the experimental condition, the experimental device and numerical setting is presented in Figure 6. The thrust and power obtained by experiment are 31.6N and 3760.5W respectively, and the relative error is no more than 10% comparing with the CFD simulation results of 30.94N and 3622.5W, which helps verify the accuracy of CFD simulation.
3.1 Aerodynamic
Here, the static pressure distribution at 25%, 50% and 75% blade span percentage is selected for comparison, as shown in Figure 7.

According to the figure, the error of pressure distribution obtained by the panel-vortex particle method and FVM is as follows: the pressure distribution calculated by hybrid method of suction surface and pressure surface is relatively high. The pressure of suction and pressure surface at the tail edge is not same, and the static pressure of suction surface of hybrid method is about 2000Pa lower than FVM. The specific reasons are (1) The high pressure distribution of suction and pressure surface is due to the weak wake induction of vortex particles in the surface element vortex particle method, which leads to the low axial inlet velocity, and the angle of attack at the inlet of blade is too big. Then, the phenomenon of insufficient pressure expansion on suction surface appears, and the velocity in blade channel is lower and static pressure on pressure surface is higher than the reality. (2) The difference of suction and pressure pressure on the tailing edge is caused by the strong induction of the equivalent vortex at the tailing edge, and the induced velocity on the suction surface and pressure surface of adjacent blade is too large, which leads to the calculation error of pressure distribution.

3.2 Aeroacoustics
The static pressure, grid area, unit normal vector and element node coordinate data are obtained by using the panel-vortex particle method and the finite volume method respectively. The acoustic characteristics of fans with the incoming duct flow condition are obtained by Goldstein pipe acoustic model. The rotating speed selected here is still 7700rpm, and the inlet speed is 30m/s. Firstly, the frequency-domain distribution characteristics of pressure fluctuation are analyzed. The time-domain pressure at a certain point on the blade surface is taken here. After Fourier transformation, it can be obtained:
From the Figure 8, there are higher peaks of blade surface pressure at the harmonic frequency of rotation frequency, and the amplitudes of these peaks at the first two harmonic frequencies are far more than other peaks, but the peaks gradually decrease at higher harmonic frequencies. There are many sawtooth between 2BPF and 5BPF. The sawtooth amplitude is low and does not show a regular distribution range, which is considered as the numerical calculation error. Because 3BPF is submerged by noise, the sound pressure distribution of ducted fan under 1BPF and 2BPF is analyzed. The observation point is set at the center of the origin and the angle interval is 5°, and the radius is 0.5m:

![SPL distribution phenomenon of duct fan](image)

**Figure 10 SPL distribution phenomenon of duct fan**

It can be seen that the noise sound pressure level of ducted fan obtained by using panel vortex particle method is larger. Under 1BPF and 2BPF, the minimum error of sound pressure level is 4.16% and 11.55% respectively, and the error of pressure distribution calculated by panel-vortex particle method results in the difference of directivity distribution of sound pressure is about 10°.

### 3.3 Further analysis

The potential flow method mainly considers the velocity circulation when dealing with the lifting body. The vortex particle method is actually a discrete description of the wake vorticity of the lifting body, and the error of the hybrid method can be seen as the error of the wake vorticity. Based on Joukowski’s lift theorem (Katz et al., 2001), the velocity circulation is equivalent to the lift. Therefore, the lift of unit length section at different blade span is analyzed below.

\[
F = -\rho U T
\]  

(18)

In order to obtain the thrust distribution, the linear density of the force at different height sections of the blade is needed. The thrust of the grid near the section can be obtained by integrating the normal vector, static pressure and grid area at the grid node, then the area and length can be integrated respectively to obtain the grid height, and finally the force of normalization processing on the grid height can be obtained:

\[
F_i = \frac{\sum_{j=1}^{n} \rho \mathbf{s} \cdot \mathbf{n}_j}{\sum_{j=1}^{n} l_j}
\]  

(19)

Where i and j represent the direction of component force and the order of mesh(panel) respectively, and n represents the total number of mesh(panel) elements at the section. The velocity circulation distribution at different blade span is analyzed:

![Thrust distribution of different elementary stage](image)

**Figure 11 Thrust distribution of different elementary stage**

As can be seen from the figure above, the total thrust of panel vortex particle method fluctuates around the thrust from FVM method with the increasing of blade span percentage. At the root of the blade, the thrust of panel-vortex particle method is smaller. However, the thrust of the panel-vortex particle method increases abruptly at the tip of the blade.

According to the above phenomenon, the induction of tip vortex and duct has a great influence on the calculation results in the panel-vortex particle method. In fact, the sudden increase of tip thrust is also the sudden increase of velocity...
circulation at the tip position, and the wake strength error of vortex particles increases, which leads to the large error of sound source distribution.

### 3.4 Computation cost

The computational time needed will be compared here to acquire an intuitive judge of the high numerical efficiency of PVM, and the CPU type is Intel(R) Xeon(R) Gold 6132 CPU @ 2.6GHz:

<table>
<thead>
<tr>
<th>Table 3 Time cost comparison of PVM and FVM</th>
<th>PVM</th>
<th>CFD(FVM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel-Vortex Particle/Mesh quantity</td>
<td>34400-13500</td>
<td>541W</td>
</tr>
<tr>
<td>CPU quantity for parallelization</td>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>RAM requirement</td>
<td>36G</td>
<td>16G</td>
</tr>
<tr>
<td>Time cost per step</td>
<td>32s</td>
<td>14s</td>
</tr>
</tbody>
</table>

According to the value given above, the PVM method requires more memory to store the panel and vortex particle data, and the array vectorization is used to speed up the potential matrix calculation. For the time cost per core, the PVM is slightly more effective than FVM.

However, the time cost will increase in a manner of square relating to the quantity of potential element, and the parallelization of FVM is more effective based on calculation of multiple meshes simultaneously. If the time step decreases to do unsteady simulation more precisely, the vortex particle quantity will be increased to keep the wake long enough, and then the time and memory cost of PVM will exceed FVM.

### 4 Discussion and Conclusions

In this paper, the panel method, vortex particle method, panel-vortex particle hybrid method and the aerodynamic-acoustic coupling method are introduced. The panel-vortex particle hybrid method and FVM method are combined with the duct acoustic model to predict the sound pressure level of the duct fan. The differences of results between these two methods are compared and analyzed based on the flow mechanism.

The pressure distribution of the hybrid method is higher than that of the suction surface and the pressure surface, and the pressure on the suction and pressure surface at the casing edge is not the same. This is because the wake strength of vortex particles is low, which leads to the low axial inlet velocity, and then the inlet angle of attack of blade is too large under the same rotating speed, and the pressure expansion effect cannot be produced on the suction surface.

According to the velocity circulation, it is found that the hybrid method will increase suddenly due to singularity when dealing with the tip area, which fully shows that the induced effect of channel will have a great influence on the flow field. In addition, the tip thrust is too high, which results in the relative error of the overall blade thrust reaching 5.36%.

From the results of the developed acoustic model, the hybrid method can obtain a sound pressure level distribution which is basically similar to FVM. The minimum error of sound pressure level is 4.16% at 1BPF, 11.55% at 2BPF, and 10 degrees derivation angle for the the minimum value of sound pressure level.

### References


