BLADE TIP SIGNAL ANALYSIS OF AN AXIAL COMPRESSOR

Ruize Xu  
School of Energy and Power Engineering, Beihang University  
xreese@foxmail.com  
Beijing, China

Jia Li  
School of Energy and Power Engineering, Beihang University  
brucelee@buaa.edu.cn  
Beijing, China

Xu Dong  
School of Energy and Power Engineering, Beihang University  
buaadongxu@buaa.edu.cn  
Beijing, China

Dakun Sun  
School of Energy and Power Engineering, Beihang University  
sundk@buaa.edu.cn  
Beijing, China

Xiaofeng Sun  
School of Energy and Power Engineering, Beihang University  
sunxf@buaa.edu.cn  
Beijing, China

Congcong Chen  
School of Energy and Power Engineering, Beihang University  
chencongcong@buaa.edu.cn  
Beijing, China

ABSTRACT

Experiments were conducted on a low-speed compressor, and a set of high response transducers were mounted in the casing to collect the over-tip signal. The auto-correlation measure results show that Rs at the first half of the blade will degrade before stall. According to the phase-locked pressure contour and RMS contour at design point and near stall, the disturbance in the blade passage is caused by separation vortices, and it converges upstream from mid chord to the leading edge (LE) of the rotor and finally leads to stall. The Proper Orthogonal Decomposition (POD) can divide the pressure signal into different POD modes. The first two POD modes correspond to the blade loading, and the energy proportion of these two modes is reduced from 97% at the design point to 94% near the stall. If the proportion threshold is set at 95%, POD can generate a warning signal 350 revolutions before stall.

INTRODUCTION

Rotating stall and surge are two main unstable phenomena of the aero-engines that seriously threaten aircraft security. In most cases, the rotating stall leads to or occurs before the surge. Detecting an imminent rotating stall accurately and providing sufficient warning time can prominently improve the safety of the aero engines. There are two typical stall inceptions in the axial compressors. One is a two-dimensional, long-scale wavelike disturbance called modal stall inception (McDougall et al., 1990). The other is spike-type inception, a three-dimensional, small amplitude disturbance (Day, 1993). Numerous studies have been made to detect an impending stall, and techniques such as spatial Fourier analysis (McDougall et al., 1990), traveling-wave-energy analysis (Tryfonidis et al., 1995), cross-correlation (Li, 2016) have been developed. Most of these techniques attempt to analyze the increasing unsteadiness of the signal before stall. However, hardly has anyone linked these techniques to the physical mechanism behind the stall evolution. As yet, a technique applicable for all compressors remains to be found.

Based on aero-acoustic theory and Kutta-Joukowski theorem, Li et al. connected the unsteady vortices shedding from the tip region that occurs before stall with the rotor blade loading (Li et al., 2016). He used auto-correlation measure to the dynamic pressure signal over the blade tip to measure the change of blade loading during the throttling and achieved a warning signal seconds before stall. Dong et al. applied this technique on a single-stage compressor and realized online control with a stall pressure-suppressed casing treatment (Dong et al., 2019).

The Proper Orthogonal Decomposition (POD) is a modal decomposition method for fluid dynamics (Taira et al., 2017). The purpose of POD is to find a set of orthogonal modes that optimally capture the energy of fluctuant flow data. The modes determined by POD are based on the spatial correlation of the data at different positions. Inspired by the auto-correlation technique, POD may also be a valuable technique for stall warning. Furthermore, POD can analyze
simultaneously analyze multiple sets of data from different sensors. This feature makes it possible to evaluate the stability of the whole compression system even under the condition of inlet distortion, which is infeasible for those methods only using one or two sensors.

This paper is organized as follows: A short review of previous studies is in Sec. 1. Sec. 2 gives the information on experimental facilities and measurement. A detailed introduction of auto-correlation measure and POD is also in this section. The results of auto-correlation and POD and relative discussions are provided in Sec. 3. Finally, key conclusions are listed in Sec. 4.

EXPERIMENTAL FACILITIES AND SIGNAL PROCESSING

The low-speed compressor (LSC)

The experiments were performed on a low-speed compressor (LSC). It contains an inlet duct, a one-stage rotor with one stator, an outlet duct, and a throttle comprising a fixed cone and movable annular sleeve (Fig. 1). The rotating speed of this compressor can vary from 0 to 3000rpm. The basic design parameters of the compressor are listed in the table. A metal plate is installed behind the inlet total pressure tube to simulate a non-uniform inlet condition of the compressor. In this study, the plate covers 10% of the compressor's inlet area. The total-to-static pressure rise characteristic of the compressor under uniform flow and distortion is shown in Fig. 2.

**Fig. 1 Single-stage axial compressor**

**Table 1 Design Parameters of the two-stage compressor**

<table>
<thead>
<tr>
<th>Geometrical Parameter</th>
<th>Rotor</th>
<th>Stator</th>
<th>Aerodynamic Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of blades</td>
<td>20</td>
<td>27</td>
<td>Design speed</td>
</tr>
<tr>
<td>Tip diameter</td>
<td>600 mm</td>
<td>600 mm</td>
<td>Mass flow</td>
</tr>
<tr>
<td>Hub-tip ratio</td>
<td>0.577</td>
<td>0.67</td>
<td>Total pressure ratio</td>
</tr>
<tr>
<td>Aspect-ratio</td>
<td>0.956</td>
<td>1.18</td>
<td>Design efficiency</td>
</tr>
</tbody>
</table>

**Fig. 2 Total-to-static pressure rise characteristics of the compressor**
Measurement system
In the experiments, the measurement system contains steady characteristic measurement and dynamic pressure measurement. The steady measurement consists of total pressure sensors in the inlet and exhaust duct and the static pressure sensors placed on the casing wall of the inlet and exhaust duct to get the characteristic parameters of the compressor. There are eight fast response pressure transducer groups (A-H) distributed uniformly around the casing. In each group, there are 14 Kulite transducers (PA1-PA14) flush-mounted chordwise from 2.4cm upstream to the rotor leading edge (LE) to the trailing edge (TE) to collect the pressure signal over the rotor blades (Fig. 3). The transducers’ sampling rate is 40 kHz, giving approximately 40 samples per blade passage at a rotor speed of 3000rpm (100% design speed). A Hall sensor is installed on the shaft to give a pulse signal at a specific circumferential position in each revolution. Then, the phase-locked pressure contour and root-mean-square (RMS) contour are calculated from the signal of 50 revolutions.

Fig. 3 Configuration of the transducers

Auto-correlation measure
The auto-correlation measure was first applied to stall warning techniques by Tahara (Tahara et al., 2007). It is found that the index computed from the auto-correlation measure will degrade before the spike in inception so that this method can generate a warning signal sufficiently in advance of the stall.

The auto-correlation measure is defined as follows:

\[
Rc(t) = \frac{\int_{-\text{wnd}}^{\text{wnd}} p(t)p(t-T)dt}{\sqrt{\int_{-\text{wnd}}^{\text{wnd}} p(t)^2dt \int_{-\text{wnd}}^{\text{wnd}} p(t-T)^2dt}} \quad (1)
\]

Where \(p(t)\) is the pressure signal of passing blades at time \(t\). \(T\) is the time of one shaft revolution. The DC value of the pressure is removed to ensure the signal has a zero mean. The subscribe wnd refers to the length of the signal in a calculating window. The value of wnd in this paper is set as the length of signal for three-blade pitches.

To clarify how disturbances with different frequencies influence the result of the auto-correlation measure, the Fourier expansion of the pressure signal is used here:

\[
p(t) = \sum_{\omega=\text{low}}^{\text{high}} \hat{p}(\omega)e^{i\omega t} \quad (2)
\]

where \(\hat{p}(\omega)\) is the Fourier transform of \(p(t)\). Put the Fourier expansion into the autocorrelation measure, according to the orthogonality of the Fourier series, and the result can be written as follows
The pressure signal collected on the casing wall can be decomposed into the signal $p_b(t)$ that is corresponding to steady blade loading and the unsteady disturbance signal $p_d(t)$. The frequency of $p_b(t)$ are equal to the rotor revolution frequency. As for $p_d(t)$, it is a broadband signal, so the autocorrelation measure of this signal is close to 0. Thus, the result of the autocorrelation measure turns to

$$Rc(t) = \frac{\int p(t)p(t-T)dt}{\sqrt{\int p(t)^2dt \int p(t-T)^2dt}} = \frac{\int \left( \sum_{\omega=-\infty}^{\infty} \hat{p}(\omega)e^{j\omega t} \right) \left( \sum_{\omega=-\infty}^{\infty} \hat{p}(\omega)e^{j\omega(t-T)} \right) dt}{\sqrt{\int \left( \sum_{\omega=-\infty}^{\infty} \hat{p}(\omega)e^{j\omega t} \right)^2 dt \cdot \int \left( \sum_{\omega=-\infty}^{\infty} \hat{p}(\omega)e^{j\omega(t-T)} \right)^2 dt}}$$

$$= \frac{\sum_{\omega=\infty}^{\infty} |\hat{p}(\omega)|^2 e^{j\omega T}}{\sum_{\omega=-\infty}^{\infty} |\hat{p}(\omega)|^2}$$

(3)

According to Parseval’s theorem, the expression of $Rc$ represents the percentage of power that blade loading signal $p_b(t)$ contributes to the pressure signal. When a compressor is working near the stall boundary, the intensity of flow separations and tip clearance vortices will grow, which will cause the blade loading at the tip region to reduce. Consequently, the power of the blade loading signal will decline, but the power of the disturbance signal will increase so that the $Rc$ will degrade.

**Proper orthogonal decomposition (POD)**

The Proper Orthogonal Decomposition (POD) is a helpful tool to analyze the fluctuation modes of flow fields (Taira et al., 2017). It can decompose the signal into a set of orthogonal bases and use as few bases as possible to contain a maximum of signal energy.

The POD can decompose the unsteady components $P'(x, t)$ of the pressure signal $P(x, t)$ collected from different positions on the casing into the following manner

$$S(x, t) = P(x, t) - \bar{P}(x, t) = \sum_i a_i(t) \phi_i(x)$$

(5)

Where $\bar{P}(x, t)$ is the temporal mean of $P(x, t)$, and $\phi_i(x), a_i(t)$ denote the POD modes and temporal coefficients of flow fields, respectively. Vector $x$ represents the spatial portion of the dynamic pressure transducer. Here, the $S(x, t)$ can be expressed as a $m \times n$ matrix

$$S(x, t) = \begin{pmatrix} s(x_1, t_1) & s(x_2, t_1) & \cdots & s(x_m, t_1) \\ s(x_1, t_2) & s(x_2, t_2) & \cdots & s(x_m, t_2) \\ \vdots & \vdots & \ddots & \vdots \\ s(x_1, t_n) & s(x_2, t_n) & \cdots & s(x_m, t_n) \end{pmatrix}$$

(6)

Then the covariance matrix $C_{xx}$ of $S(x, t)$ can be calculated

$$C_{xx} = \frac{1}{n} S^T S$$

(7)

and the eigenvectors $\phi_i$ and the eigenvalues $\lambda_i$ are computed from

$$C_{xx} \phi_i = \lambda_i \phi_i, \quad \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m \geq 0$$

(8)
The eigenvalue $\lambda_i$ denotes the energy captured by the POD mode $\phi_i$. And the temporal coefficients are determined by

$$a_i(t) = \langle \mathbf{S}(x,t), \phi_i(x) \rangle_x$$  \hspace{1cm} (9)

For the pressure signal collected from the casing wall, the energy mainly centers on the part that is relative to the steady blade loading, and the first several POD modes take the dominant part. When the compressor operates near the stall boundary, the power of disturbances will grow, and the energy and, as a result, the energy of POD modes related to steady blade loading will decrease.

**RESULTS AND DISCUSSION**

**Auto-correlation results**

The auto-correlation measure is made for the pressure signal collected at circumferential location C during the throttling process from the design point to the stall point. It can be seen from Fig. 4 that the Rc at PA5-PA10 is over 0.9 when the flow rate is high. The Rc at PA10 begins decreasing with the throttling going on. Shortly afterward, a detrend of Rc emerges at PA9 to PA5 in consequence, and PA5 shows the most significant degradation of the index 100 revolutions before the stall. The phenomenon suggests that the disturbance first appeared at mid-chord (PA10) and propagated upstream to the LE (PA5). Once the disturbance arrived at the LE, it would increase blade incidence and intensified the separation.

Consequently, the disturbance grew rapidly at the LE, leading to a noticeable deterioration of Rc here. However, the Rc at upstream transducers (PA1-PA5) and transducers after the PA10 stayed at a low level and did not show significant change during the throttling process. It has been discussed before that the result of the auto-correlation measure represents the ratio of signal power contributed from steady blade loading to full signal power. Transducers PA1-PA4 is located before the rotor LE, and the pressure signal can hardly reflect the change of blade loading. Instead, these upstream transducers are more likely to be influenced by inlet flow fluctuation. Transducers PA11-PA14 are located at the second half of the blade, where the flow separation is violent. The blade loading here is not so high as that at the first half of the blade. Thus the indices at these positions stay relatively stable until fully developed stall.

![Fig. 4 Rc at different axial locations](image)

However, it remains uncertain whether flow separation vortex or tip leakage vortex caused the disturbance in the blade passage. Fig. 5 and Fig. 6 show the phase-locked pressure contours and the RMS contours measures by the transducers at
the design point and near stall. In Fig.5, it can be seen from the phase-locked pressure contour that the low-pressure core is attached to the suction side of the blade, and the trajectory of the low-pressure field lies through the blade passage. When the operating point is near stall, the pressure contour of Fig.6 shows that the low-pressure core sheds from the suction surface and moves slightly towards the LE of the adjacent blade, and the trajectory becomes more parallel to the LE of the rotor. RMS is a measure of pressure fluctuations in the blade passage. High RMS usually corresponds to the position of the shear layer between the unsteady secondary flow and the main flow. In Fig. 5, the high RMS trajectory lies along the chordwise, and this trajectory becomes parallel to the rotor LE near stall in Fig. 6. Considering that the low-pressure core moves from the suction side of the blade to LE of the adjacent blade near stall and the highest RMS constantly stays at the LE, it seems reasonable that the separation vortex causes disturbance in the blade passage. During the throttling process, the leading-edge separation causes the separation vortices at the mid chord. Then, those vortices convect upstream towards the LE of the adjacent blade, increasing the next blade's incidence causes the separation vortices in the adjacent blade passage. These vortex sheddings lead to the fluctuation of the blade loading that can be detected by auto-correlation measures.

Fig. 5 Phase-locked pressure contour and RMS contour at the design point
POD results

It is now clear that the disturbance caused by leading-edge separation mainly appears at the first half of the blade passage according to the results of the auto-correlation, so the pressure signals collected from PA5-PA10 are chosen for POD analysis. Fig. 7 compares the energy distribution of the different modes of POD at the design point and near stall. At the design point, the energy of the first two modes covers 97% of the whole signal energy, and this data declines to 94% before stall. Whereas the energy of the rest modes grows from 3% to 7%. To verify what flow phenomenon each POD mode stands for, windowed Fourier transform was applied to the POD temporal coefficients $a_i(t)$ during the throttling process. In Fig. 8, the frequency of POD temporal coefficients of the first two modes is concentrated on the BPF and multiple BPF, which suggests that these two modes are relative to the steady blade loading. The frequency components of modes three to five are much more complex, and apparent disturbance (red box) can be observed in the frequency spectrum of these modes. It is rational to speculate that the increased energy proportion of these modes is attributed to the growth of disturbance in the blade passage near the stall. In addition, the signal related to blade loading always takes a dominant part, and the disturbance is relatively tiny. This feature is also in accord with the energy proportion of the first two POD modes.

![Fig. 6 Phase-locked pressure contour and RMS contour near stall](image)

![Fig. 7 Energy distribution of different POD modes](image)

![Fig. 8 POD temporal coefficients](image)
Since the POD can separate the signal corresponding to blade loading and disturbance signal, the energy proportion of the first two POD modes may potentially be used as a criterion for stall warning. To validate this, the POD is made for every 25 revolutions during the throttling process, and the result is depicted in Fig. 9. Although there is slight fluctuation, the energy proportion of the first two modes presents a continuous detrend when the compressor is driven to the stability boundary. Once the stall occurs, the proportion begins to drop drastically below 90%. If the warning threshold of energy proportion is set as 95%, the warning signal will be generated 350 revolutions before stall, which could provide sufficient time for a control system to react.

Fig. 9 Energy proportion of the first two POD modes

The effectiveness of POD under the inlet distortion is also studied. Eight transducers are evenly mounted around the casing at axial position PA7. It can be seen in Fig. 2 that the pressure rise is significantly decreased, and the flow coefficient of the last steady operation point is bigger when the compressor works under the inlet distortion. Fig. 10 shows how the energy proportion of the first two POD modes changed during the throttling process. In the figure, the energy proportion of the first two POD modes shows a similar detrend as in Fig. 9. This stable performance of POD indicates that this technique has the potential to provide a reliable warning signal under the influence of inlet distortion and can be used for compressors in the real environment.
Fig. 10 Energy proportion of the first two POD modes under inlet distortion

CONCLUSIONS
In this study, the auto-correlation measure and POD are used to analyze the pressure signal collected from a one-stage axial compressor rotor tip during the throttling process. The key findings are summarized as:

- The first decline of Rc starts at the mid-chord of the rotor blade, and this detrend propagates upstream with the flow rate decreasing. The most significant degradation of Rc appears about 100 revolutions before stall as rotor LE.
- In this compressor, the stall disturbance is produced by leading-edge separation. The separation vortices move upstream to the adjacent blade LE and propagate around the rotor. These vortices can affect the loading of the blade and finally lead to the stall.
- POD can separate the blade loading signal and the disturbance signal. The first two POD modes are related to the blade loading, and the rest modes represent disturbance signals.
- The energy proportion of the first two POD modes declines under the uniform and distortion inlet flow when the flow rate decreases. This feature of POD can be used for stall warning. If the proportion threshold is set at 95%, a warning signal can be generated 350 revolutions before stall.

NOMENCLATURE

- $a_i(t)$: POD temporal coefficients
- $c_{xx}(x, t)$: covariance matrix of $S(x, t)$
- $R_c$: result of correlation measure
- $p(t)$: dynamic pressure signal
- $\hat{p}(\omega)$: fourier transform of $p(t)$
- $P(x, t)$: dynamic pressure signal matrix
- $\overline{P}(x, t)$: temporal mean of $P(x, t)$
- $p_b(t)$: blade loading signal
- $p_d(t)$: disturbance signal
- $S(x, t)$: difference between $P(x, t)$ and $\overline{P}(x, t)$
- $\lambda_i$: eigenvalue of the $i^{th}$ POD mode
- $\phi_i(x)$: the $i^{th}$ POD mode

ACKNOWLEDGMENTS
The research presented here is supported by the National Natural Science Foundation of China (No. 11661141020, 51576008, 51822601, and 51790514).
REFERENCES


