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Mode Shape and Eigenfrequency Analysis based on Helmholtz Equation for Thermo-acoustic Combustion Instability

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ABSTRACT

The thermo-acoustic combustion instability is a key problem in combustion that stems from the interaction between the acoustic wave and fluctuations of heat release. In this paper, the finite element software COMSOL is used to identify the frequencies and the growth rates at which thermo-acoustic instabilities are excited. The Helmholtz equation combined with the classical n-τ flame model is solved to capture the thermo-acoustic features in simple geometry combustors with no mean flow. The accuracy of this method is demonstrated by studying the thermo-acoustic in various verification cases. The effect of the flame holder on acoustic pressure field is studied. The results show that the first transverse mode is concentrated around the flame holder. The mode frequencies will change as the temperature increases in the downstream of flame holder. In addition, when the upstream temperature of the flame holder is different from downstream, the longitudinal mode will transition to transverse mode. Furthermore, combustion instability in the combustion chamber of the model rocket is studied by using this method and compared with the experimental data. The unstable mode frequencies are 1451Hz, 1184.9Hz, 1704.7Hz and 1821.5Hz, respectively. This work demonstrates that the method can be applied to predict the thermo-acoustic behavior.

Keywords: Combustion Instability, Acoustic Waves, Heat Release Fluctuation, Growth Rate, Helmholtz Equation

INTRODUCTION

Thermo-acoustic combustion instability is a resonance phenomenon that arises due to the coupling of the unsteady heat release and the system acoustics. The pressure and heat release oscillation generated in the coupling process will lead to the wear of components and shorten the operation life of combustion chamber. With the development of aero-engines, thermo-acoustic combustion instability will inevitably become the focus of combustion research. At present, the methods that have been proposed to study thermo-acoustics include low order models (LOM), large eddy simulation (LES) and finite element methods (FEM).

The basic idea of low order models (Dowling, 1995; Polifke et al., 2001; Schuermans et al., 2003) is to study the pressure oscillation by dividing the combustion system into a series of subsystems and using the mathematical transfer function matrix to connect these acoustic elements to each other, so as to provide the continuity of sound speed and pressure across each zone. Low order models need low computational cost, but it cannot model the propagation of the acoustic waves adequately for complex three-dimensional geometry. The purpose of large eddy simulation (Selle et al., 2004; Giauque et al., 2005; Staffelbach et al., 2009) is to study the phenomenon of combustion instability and matching pressure oscillation with turbulent combustion phenomena. The ability of LES in solving thermo-acoustic problems has been demonstrated (Selle et al., 2004), but this method needs a lot of numerical resources. A (The) finite element method uses acoustic models to solve acoustic problems in three-dimensional geometry (Pankiewitz and Sattelmayer, 2003; Campa and Camporeale, 2009). This method numerically solves the differential equation problem of the acoustic wave propagation with heat release fluctuations, converting it into a complex eigenvalue problem in the frequency domain and detects complex eigenfrequencies of the system to ascertain whether the corresponding mode is unstable or stable (Camporeale et al., 2011). The eigenvalue problem is nonlinear and it is solved by means of linearization under the hypothesis of small perturbations.

In this paper, COMSOL Multiphysics based on finite element method (FEM) is used to identify unstable frequency (frequencies) and analyze thermo-acoustic combustion instability, that is, to solve the acoustic governing equation coupled with combustion response function in the frequency domain. This method can calculate multiple modes at the same time, requires lower computational resources and has better flexibility in defining the geometry of the combustion chamber. The accuracy of this method is firstly demonstrated by various verification cases. The effect of the flame holder on acoustic is studied. Furthermore, this method will be applied to model rocket combustor.

GOVERNING EQUATIONS

The effects of viscosity, thermal diffusivity and heat transfer are neglected, the mean pressure is assumed uniform in the computational domain. The flow velocity is much lower than sound velocity so that the flow velocity can be generally considered negligible. Under such hypotheses, the inhomogeneous wave equation with heat release fluctuation (Dowling and Stow; 2003) is as follows:

$$\frac{1}{\bar{c}^2}\frac{\partial^2 p'}{\partial t^2} - \bar{\rho}\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla p'\right) = \frac{\gamma-1}{\bar{c}^2}\frac{\partial q'}{\partial t} \quad (1)$$

Where q' representing the fluctuation of the heat release per unit volume. In the harmonic analysis, pressure, velocity and heat release fluctuation can be represented by complex function of time:

$$p' = \mathrm{Re}[\hat{p}\exp(i\omega t)] \quad (2)$$

$$u' = \mathrm{Re}[\hat{u}\exp(i\omega t)] \quad (3)$$

$$q' = \mathrm{Re}[\hat{q}\exp(i\omega t)] \quad (4)$$

Where ω complex angular frequency, the real part of ω gives the frequency of the oscillations, $f = \mathrm{Re}(\omega)/2\pi$, while the imaginary part corresponds to the growth rate, $\alpha = -\mathrm{Im}(\omega)/2\pi$. If α is positive, the amplitude of fluctuations grows with time. Introduction (Introducing) Eq. (2) and Eq. (4) into Eq. (1), the Helmholtz equation can be obtained:

$$\frac{\lambda^2}{\bar{c}^2}\hat{p} - \bar{\rho}\nabla\cdot(\frac{1}{\bar{\rho}}\nabla\hat{p}) = -\frac{\gamma-1}{\bar{c}^2}\lambda\hat{q} \quad (5)$$

Where $\lambda = -i\omega$ is the eigenvalue. Eq. (5) shows a quadratic eigenvalue problem.

Combustion Response Models

Unstable heat release fluctuation is an important factor causing combustion instability. In order to solve the problem of heat release fluctuation, the combustion response function is introduced.

Based on the Crocco's $n-\tau$ model (Crocco, 1969), there is a time delay τ between the unsteady heat release fluctuation and the pressure fluctuation, $q'/\bar{q} = n p'(t-\tau)/\bar{p}$. The wave equation with combustion response function in the frequency domain is

$$\frac{\lambda^2}{\bar{c}^2}\hat{p} - \bar{\rho}\nabla\cdot(\frac{1}{\bar{\rho}}\nabla\hat{p}) = -\frac{\gamma-1}{\bar{c}^2}\cdot\lambda\cdot n\cdot\frac{\hat{p}}{\bar{p}}\cdot e^{-i\omega\tau}\cdot\frac{\bar{q}}{V} \quad (6)$$

Where n is interaction index that quantifies the intensity of heat release, $\bar{q}$ is the mean heat release in Watt, V is volume of heat release zone. The combustion response model is incorporated into the Helmholtz equation solver as a part of the monopole source, and the combustion is considered by the pressure-time lag model.

NUMERICAL SIMULATION OF MODEL COMBUSTORS

Verification tests

The longitudinal combustor without heat release

A simple longitudinal combustor with a length of 0.805m and a radius of 0.021m is considered (Sisco et al., 2006). The flow parameters from the experiment are as follows: temperature T=933.5 K, density ρ=6.295 kg/m3, pressure p=2.21 MPa, speed of sound c=667.8 m/s. Different boundary conditions of duct end are used for analysis, and the analytical solutions are as follows:

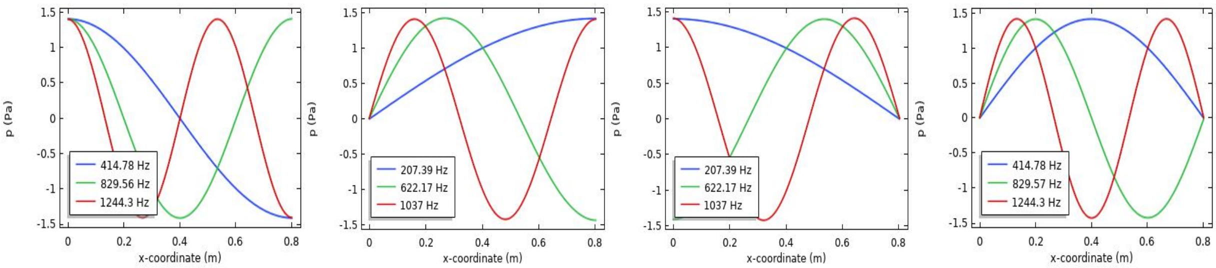
Closed - Closed or Open - Open: $$f = \frac{nc}{2l} \quad (7)$$

Closed - Open or Open - Closed: $$f = \frac{(2n-1)c}{4l} \quad (8)$$

The results are shown in Table 1. By comparison, the resonant frequencies obtained by the current approach are in good agreement with analytical solution. Figure 1 shows the first three order mode shapes of straight duct with different boundary conditions. For each longitudinal mode, acoustic pressure nodes exist in open boundary, acoustic pressure antinodes exist in closed boundary and the mode number is equal to the number of pressure antinodes or nodes. Since the system has neither loss mechanism nor heat release, it is expected that the pressure fluctuation has neither decay nor grow. The same phenomenon has been observed in the results obtained by the current method.

Table 1. Resonance frequency under various boundary conditions

Inlet	Exit	Frequency (Hz)	
		Helmholtz	Analytical
Closed	Closed	414.78	414.78
Closed	Open	207.39	207.39
Open	Open	414.78	414.78
Open	Closed	207.39	207.39



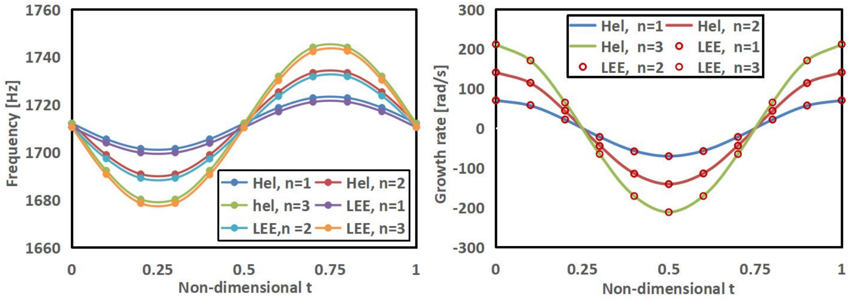
(a) close - close (b) open - close (c) close-open (d) open - open

Figure 1 first three order mode shapes of straight duct under various boundary conditions

The longitudinal combustor with heat release

The longitudinal combustor is 0.635m long with a compact flame in the middle of the domain. The flow properties in the domain are as follows: T = 2544.8 K, ρ = 2.179 kg/m^3, p = 2.16 MPa, c = 1087.2 m/s. The heat release from experiment is 711KW (Sisco et al., 2006).

The frequency and growth rate of 2L mode with different n and τ are shown in Figure 2 and compared with the results of Linearized Euler equations (LEE) (Tamanampudi and Anderson, 2015). The results are so close to those obtained from LEE that proves the reliability of this method. The non-dimensional t represents the time delay τ normalized by the oscillation period T. The growth rate is positive for 0< t < 0.25 and 0.75 < t <1, while the growth rate is negative when 0.25 < t < 0.75. Unstable heat release is in phase with pressure fluctuation, the amplitude of fluctuations increases gradually and the system oscillation tends to be strong from the Rayleigh Criterion (criterion). Obviously, the system will become unstable with 0< t<1/4 and 3/4 < t <1.



(a) Frequency (b) Growth rate

Figure2 Variation of frequency and growth rate of 2L mode with n and t

Application of combustion response model

Effect of flame holder on acoustics

In this part, the effect of a flame holder and temperature gradient on acoustic mode shapes are studied. The model used the test section to stabilize flame of the Combustion Wind Tunnel Facility at the University of Cincinnati (Shaw et al., 2018), as shown in Figure 3. The length of the model is 1.6281m, with open inlet and exit. Since there is a temperature gradient when the flame is stable in the test section, a separate "flame zone" is defined in this model.

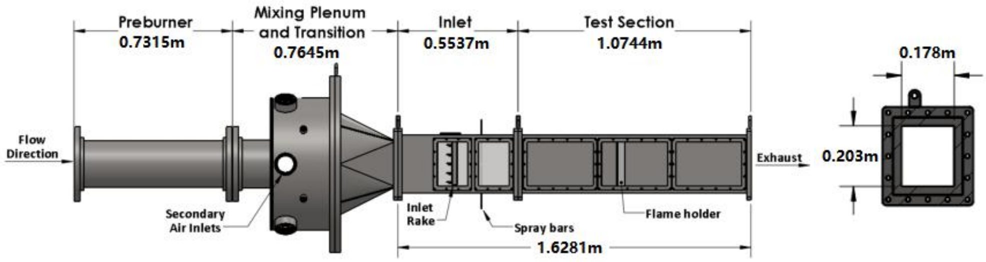
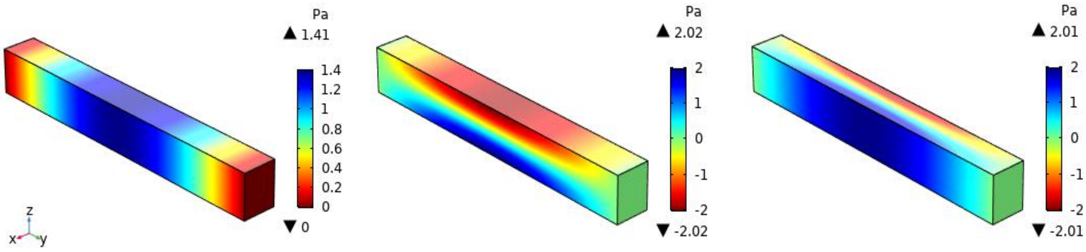


Figure 3 Combustion Wind Tunnel Facility (Shaw et al., 2018)

Figure 4 and Figure 5a show the mode shapes of rectangular tube and rectangular tube with flame holder. There is no significant change in the longitudinal mode and transverse mode along the height, so the first two modes are not listed in Figure 5a. However, the transverse mode along the width changes and concentrates on both sides of the flame holder to form a symmetrical high pressure region that is 180° out-of-phase.

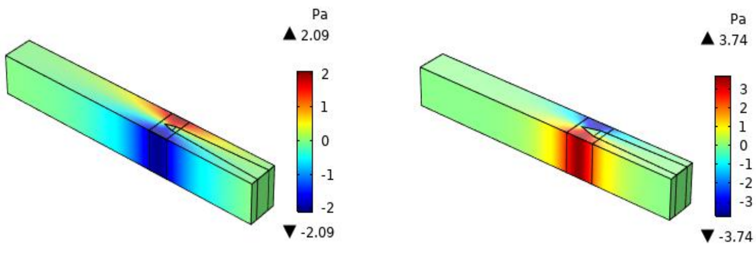
Transverse mode oscillation is excited as the result of periodic transport of fuel into the hot wake of the flame holder and the high-frequency oscillation is accompanied by vortices shedding periodically (Rogers and Marble, 1956). While vortices shedding is due to the formation of boundary layer on the flame holder surface (Bearman, 1984). The high pressure region on both sides of the flame holder disturbs the boundary layer, resulting in periodic vortices shedding and fuel transportation, which lead to the pulsation of heat release. The coupling of heat release pulsation and combustor longitudinal mode results in combustion instability (FU (Fu) et al., 2015). Therefore, the coupling between the heat release pulsation and the transverse mode may lead to the generation of high frequency oscillation. If the heat release pulsation is in phase with the acoustic pressure fluctuation, the instability will be amplified.

Further study on enlarging flame holder, the transverse mode is shown in Figure 5b. The mode distribution around the flame holder is more concentrated compared with Figure 5a, and acoustic pressure is significantly increased. This will reduce the interaction between the acoustic field and the flame surface, but the increase of acoustic pressure will more easily cause combustion instability and the occurrence of combustion oscillation will be closer to the flame holder.



(a) longitudinal: f=105.4Hz (b) transverse: f=851.05Hz (c) transverse: f=970.88Hz

Figure 4 Acoustic pressure for the first modes of rectangular tube

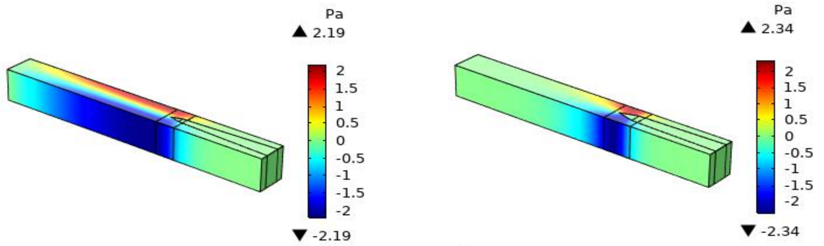


(a) f=925.49Hz (b) changed flame holder: f=925.78Hz

Figure 5 Acoustic pressure for the first transverse modes along the width with flame holder

The temperature of "flame zone" in the model is set to 1600K, and the temperature in other zones is unchanged. The first transverse mode is shown in Figure 6a. As temperature in the flame domain grows, frequency increases. Figure 6b shows the first transverse mode of rectangular tube with enlarged flame holder. The effect of the temperature gradient is to concentrate the longitudinal component of the transverse mode on the upstream of the flame holder, while the longitudinal component is concentrated around the flame holder by enlarging the flame holder. It can be seen that enlarging the flame holder will lead to combustion instability closer to the flame holder.

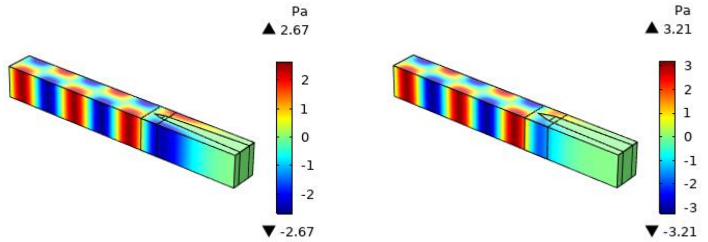
The fluid flow disturbance will affect the heat release rate, which leads to acoustic pressure fluctuation. Similarly, the acoustic pressure fluctuation will also affect the fluid flow. Therefore, the acoustic pressure upstream of the flame holder will inevitably affect the fuel flow and lead to combustion instability. In addition, the high pressure near the flame holder further indicates that combustion instability is prone to occur near the flame holder.



(a) f=969.2Hz (b) changed flame holder: f=889.65Hz

Figure 6 Acoustic pressure for the first transverse modes along the width with temperature gradient

It is also worth studying the mode transition in the system with flame holder and temperature gradient. The transition from the upstream longitudinal mode to the transverse mode downstream of the flame holder is shown in Figure 7a. As flame holder enlarges, the symmetrical mode on both sides of flame holder become more concentrated in Figure 7b, which reduces the interaction between downstream acoustic field and flame surface. In addition, the acoustic pressure also increases. The pressure wave impinges on the front of the flame holder, resulting in the symmetrical distribution of the modes on both sides of the flame holder, which is likely to produce vortices in the x- and z- directions.



(a) f=1287.6Hz (b) changed flame holder: f=1279.3Hz

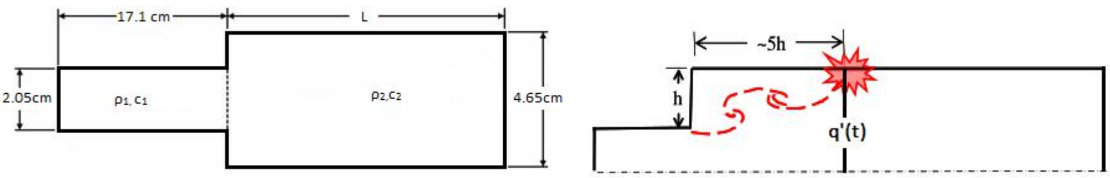
Figure 7 Mode transition with temperature gradient

Application to a model rocket combustor

The effects of combustion chamber length and heat release rate on combustion instability are studied in this part. The model (Miller et al., 2005) is divided into two parts, as shown in Figure 8a. Assuming that the head of the oxidizer post and the nozzle are rigid walls, so as to provide a highly efficient reflecting surface, which are consistent with the setting of the high reflection short nozzle used in the experiment.

The acoustic resonance frequencies calculated by acoustic model are compared with the unstable frequencies measured in the experiment are shown in Table 2. The deviation between the resonance frequencies predicted by the coupled acoustic model and that measured in the experiment is within 6%, while the deviation of the resonance frequencies predicted by the acoustic model (Eq. (7)) is within 10%. Obviously, the resonant frequency predicted by the coupled acoustic model is more accurate. The frequency deviation between the coupling model and the acoustic model indicates that the oxidizer post has an effect on the acoustic characteristics of the system. In addition, the mode transition from low order to high order with the increase of combustor length is observed.

Mode frequencies are not integer multiples of each other for the same length combustor, such as the first 4L mode frequencies of the coupled system are 871.65, 1184.9, 2150.6 and 2871.9Hz with 50.8cm long combustion chamber. The integer multiple relationship of each mode frequency is the result of the model energy transfer caused by the linear acoustic effect of the system. The current phenomenon can be considered as a nonlinear acoustic effect.



(a) Computational domain (b) The location of vortices impingement

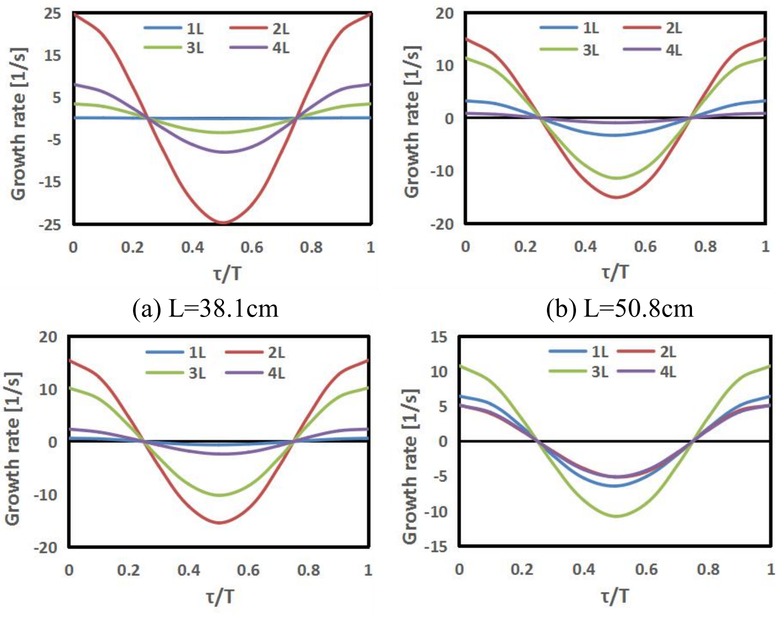
Figure 8 Computational domain of experimental rig and the location of vortices impingement

Table 2. Comparison of frequency between calculated modes and measured frequencies

Test	Chamber Length	O/F	Pc	Density (Chamber)	Unstable Frequency (Experiment)	Frequency (only acoustic)	Deviation	Frequency (Coupled acoustic)	Deviation
	cm		MPa	kg/m3	Hz	Hz	%	Hz	%
10D1	25.4	6.3	2.38	2.27	stable	-	-	-	-
15D1	38.1	6.2	2.21	2.10	1502	1430.4-1L	4.77	1451-2C	2.47
20D1	50.8	6.5	2.21	2.10	1184	1072.8-1L	9.39	1184.9-2C	0.07
25D1	63.5	6.5	2.14	2.04	1709	1716.5-2L	0.44	1704.7-3C	0.25
35D1	88.9	6.3	2.18	2.08	1721	1839.1-3L	6.86	1821.5-4C	5.8

Assuming that acoustically induced vortices shedding is the controlling mechanism, vortices impinges on the wall of the combustor and produces unstable heat release. The location of impingement point is shown in Figure 8b. The compact unsteady heat is 1.8 MW (Portillo et al., 2007).

Figure 9 shows the variation of the first 4L mode growth rate. The growth rate is positive for $0<\tau/T<0.25$ and $0.75<\tau/T<1$, which is in line with Rayleigh Criterion (criterion). In the figure, each combustion chamber with different lengths has its own mode that the growth rate is the largest. For example, the growth rate of 2L mode is the largest for 38.1cm long combustion chamber. The maximum growth rate explains why this mode is more likely to be unstable, and the corresponding unstable mode will transition from a low mode to a high mode as the length increases. Figure 10 shows the variation of frequencies and growth rates with the length of combustion chamber at n=1 and $\tau=0$. In Figure 10a, the 2L mode growth rate decreases as the length increases, while the growth rates of the 3L and 4L mode are gradually higher than that of the 2L mode and 4L mode growth rate emerges a linear growth trend. Obviously, the longer the combustor is, the more unstable the higher mode is. The mode transition from low order to high order with the increase of combustor length is also observed from Figure 10b.

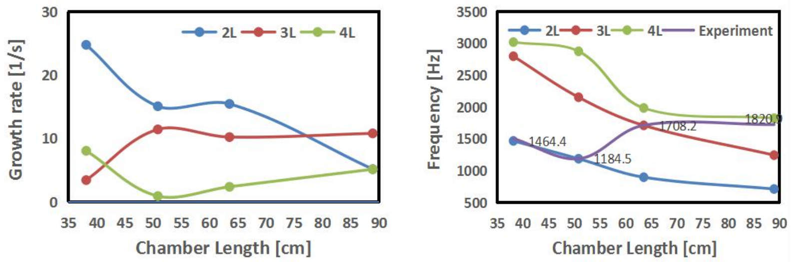


(a) L=38.1cm (b) L=50.8cm

(c) L=63.5cm (d) L=88.9cm

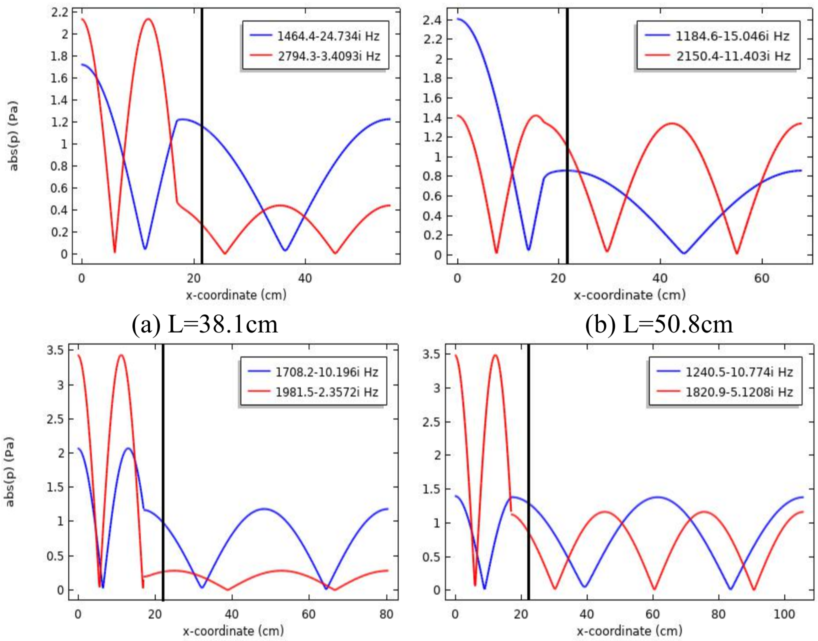
Figure 9 The variation of growth rates of firs 4L mode for different combustor length with n=1

The longitudinal mode shapes of combustion chamber are shown in Figure 11. The compact heat release is added at the black solid line. The larger the pressure amplitude in the combustion chamber is, the larger the growth rate is. In Figure 11b, the 3L mode (2150.4 Hz) is more unstable due to the high pressure in 50.8cm long combustion chamber. However, the test 20D1 shows that the unstable frequency is 1184 Hz, which is very close to the 2L mode frequency (1184.6 Hz). This may indicate that the combustion process is more easily coupled with the weaker 2L acoustic mode. In addition, the highest pressure amplitude occurs in the oxidizer post, and the pressure in the oxidizer post is always higher than that of the combustion chamber, which illustrates that the combustion system coupled with oxidizer post is more prone to combustion instability.



(a) growth rate (b) Frequency

Figure 10 The variation of frequency and growth rate at n=1 and τ= 0



(a) L=38.1cm (b) L=50.8cm

(c) L=63.5cm (d) L=88.9cm

Figure 11 Acoustic pressure distribution in the combustion chamber

CONCLUSIONS

The method used in this paper has been applied to three-dimensional geometry and even more complicated configurations with lower cost, and the frequency and growth rate can be accurately captured through analysis. This work demonstrates that COMSOL can be applied to predict the thermo-acoustic behavior.

In this paper, the eigenfrequency and the thermo-acoustic combustion instability analysis are carried out by using the Pressure Acoustic Module of COMSOL Multiphysics based on FEM. Firstly, the current method is applied to a simple longitudinal combustor and the results are in good agreement with results obtained by different analysis tools. The effect of flame holder and temperature gradient on acoustics are studied in the second step. The acoustic field is concentrated on both sides of the flame holder with the addition of the flame holder, forming a 180° out-of-phase high pressure region, which can help explain the vortices shedding during high-frequency oscillation. The longitudinal component of the transverse mode is concentrated upstream of the flame holder with temperature gradient, which breaks the longitudinal symmetry of the mode. However, when the flame holder is enlarged, the transverse mode is more concentrated on both sides of the flame holder and the pressure increases. The acoustic pressure upstream and downstream of the flame holder will affect the combustion instability. Finally, the model rocket combustor is studied. Frequency obtained by the current method is very close to the experimental results without heat release and the mode transition from low order to high order with the increase of combustor length is observed. When there is heat release, the larger the growth rate is, the larger the pressure amplitude in the combustion chamber is, which explains why the corresponding mode is more likely to be unstable. Moreover, the higher mode is more likely to be unstable with the increase of combustor length and the combustion system coupled with oxidizer post is more prone to combustion instability.

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