EFFECT OF LEADING-EDGE/TRAILING-EDGE GEOMETRY ON UNCERTAINTY AERODYNAMIC PERFORMANCE OF A 2D LOW-PRESSURE TURBINE BLADE

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ABSTRACT

Inevitable geometric variations in manufacturing process will lead to variations of aerodynamic performance of turbine blades, and uncertainty design considering this influence is significant for the robustness of the actual performance. In this paper, the uncertainty aerodynamic performance, which considers the influence of the same geometric variations, under different design conditions of leading-edge/trailing-edge geometry was compared and analysed in detail. The results showed that the actual performance is mainly affected by the leading-edge geometry, and the influence of trailing-edge geometry is relatively small. The design value and mean value of total pressure loss are mainly negatively affected by the short radius and the wedge angle of leading-edge ellipse, and positively affected by the radius of trailing edge, respectively. Similarly, the standard deviation of total pressure loss is mainly negatively affected by the short radius and the wedge angle, and positively affected by the long/short radius ratio of leading-edge ellipse, respectively. Detailed analysis explained the mechanisms, and the guidance for the corresponding uncertainty design was derived.

INTRODUCTION

Turbine blade is one of the important parts of aeroengine, and its aerodynamic performance will have a significant impact on the performance of turbine component and even the whole engine. However, considering the geometry complexity, processing difficulty and working environment harshness, the actual blade geometry is inevitably different from the nominal state. As a result, the actual performance will deviate from the design state and have great fluctuation (Balan et al. 1984). One effective method is integrating the influence of uncertainty into the entire design system, and taking the robustness of actual performance as an important evaluation index. The effective uncertainty design will reduce the influence of geometry uncertainty and is of great significance for the improvement of the actual comprehensive performance of the turbine blade (Zang et al. 2002; Zou et al. 2018).

At present, the main design method considering the influence of geometry uncertainty of compressor/turbine blades is robust optimization of existing blade profiles (Wagner et al. 2015). Through the optimization taking the actual uncertainty performance as the object, such as mean values and standard deviations under the influence of geometric variations, some optimized blade profiles less affected by the geometric variations will be obtained. Kumar (2006) optimized a 2D compressor fan profile considering the geometry uncertainty due to corrosion by the combination of the NSGA-II optimization method and the Kriging surrogate model, and the optimized profile showed better robustness and less sensitive to the geometric uncertainty. Ma (2017) optimized a certain 2D compressor blade geometry by combining the NSGA-II optimization method and the NIPC surrogate model, and standard deviations of the loss and static pressure ratio of the new blade were effectively reduced, while the mean values were basically unchanged. Kamenik (2018) proposed a robust optimization method based on sensitivity analysis and taken a turbine rotor profile as example, and the actual isentropic efficiency considering the geometry uncertainty was effectively improved, which proved the effectiveness of the optimization process. Currently, most researches on blade profile robustness optimization focus on the optimization
methods itself, and there is little analysis on the mechanism of improving the performance robustness based on the optimization results. The guiding significance for considering the influence of geometric uncertainty in the design based on these researches is weak, and the researches on uncertainty design integrating the influence of geometric variations but not robust optimization are needed.

The local geometry of the leading edge and trailing edge may have significant effects on performance parameters such as aerodynamic loss or efficiency of compressor/turbine blades. Zhang (2012) pointed out that the geometry of the leading edge of turbine blade will have a significant impact on the flow and loss of the boundary layer by affecting the leading-edge pressure spike and separation bubble. Xu (1988) compared the aerodynamic losses of transonic turbine cascades with different geometries by experiment and pointed out that the wake loss at different Mach numbers is basically proportional to the trailing-edge thickness. Zhou (2013) showed that, for turbine blade, when the trailing-edge thickness increases, the area with high base pressure near trailing edge increases, while the overall back-pressure decreases and the downstream diffusion increases, resulting in an increment in the total aerodynamic loss. Considering the comprehensive performance under the influence of actual geometric variations, the geometry of the leading edge and trailing edge will also have a great impact, which has been seldom paid attention to in the existing research and will be focused on in this article.

In this paper, through parametric blade profiling method and uncertainty calculations, the uncertainty performance of turbine blades with different design conditions of leading-edge/trailing-edge geometries was compared in detail. Combined with sensitivity analysis, the influence of leading/trailing edge geometry was compared and analysed. Furthermore, through single-variable analysis, the physical mechanism of the influence of each geometry parameter on the performance was studied, and the guidance for the corresponding 2D uncertainty design in the actual was obtained.

BLADE PROFILING AND NUMERICAL SIMULATION METHODOLOGY
The parametric blade profiling method and numerical simulation method are the basement of the subsequent analysis.

Parametric blade profiling method
A parametric blade profiling method was adopted in this paper for the 2D turbine blade parametric profiling. Twelve parameters, including axial/tangential chord length $b_5/b_6$, pitch $t$, throat width $a$, leading/trailing edge metal angles $\beta_1/\beta_2$, short/long radius of leading-edge ellipse $r_{11}/r_{12}$, radius of trailing edge $r_2$, leading/trailing edge wedge angles $\omega_{1}/\omega_{2}$ and unguided turning angle $\delta$, are selected as the basic parameters. As shown in Figure 1, the blade profile is divided into five sections: leading-edge ellipse, trailing-edge small circle and suction front/rear section and pressure side by taking five key points, including the connecting points of the leading/trailing edge and the profile lines on both sides, and the projection point of the throat on the suction side. The curves 12, 23 and 45 are all cubic polynomials, and the curves 12 and 23 are connected with spline transition to ensure curvature continuity. The advantages of this profiling method are that the design parameters are independent of each other, intuitive and clear, and the physical meaning is clear, which is convenient for subsequent analysis.

In this paper, the T106D-EIZ blade profile (Stadtmüller, et al. 2001) was selected as a typical low-pressure turbine blade template profile, and is used to provide typical profile and working condition. With reference to the T106D-EIZ blade profile, the obtained profile is shown in Figure 2, in which the chord length was set to 30mm to simulate the real size of the low-pressure turbine blade. It can be seen that the obtained profile is basically in good agreement with the T106D-EIZ profile, and there is only some deviation in the rear section of the pressure side. The effectiveness of the profiling method will be proved in the following description. The obtained profile will be used as the Reference profile for the subsequent analysis.
Numerical simulation method

The boundary conditions referred to the Condition 2 (Stadtmüller, et al. 2001), and the outlet Reynolds number and Mach number based on isentropic expansion are \(2.0 \times 10^5\) and 0.59 respectively. The specific parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Inlet total temperature/K</th>
<th>Inlet total pressure/Pa</th>
<th>Outlet static pressure/Pa</th>
<th>Inflow angle/°</th>
<th>Inlet turbulence/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>287</td>
<td>5.7(\times 10^4)</td>
<td>4.5(\times 10^4)</td>
<td>37.7</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Considering that the research object is a single cascade, the total pressure loss coefficient \(L\) and the outflow angle \(\beta_o\) were chosen to characterize the aerodynamic performance. And the \(L\) is defined as:

\[
L = \frac{P_o - P_{so}}{P_{in} - P_{so}}
\]

(1)

where \(P_o\) and \(P_{so}\) are the total pressure at the inlet/outlet of the blade passage, and \(P_{so}\) is the static pressure at the outlet of the blade passage. The Reynolds Average Navier-Stokes (RANS) solver MAP (Ning, 2014) is applied to simulate the aerodynamic performance, where the Shear Stress Transport (SST) turbulence model combined with the \(\gamma-Re_{th}\) transient model is employed. Referring to the experimental data (Stadtmüller, et al. 2001), the scaled T106D-EIZ blade is used to verify the simulation results. According to the experiment instructions, the inlet flow angle is adjusted to 42 ° and other parameters are consistent with the above table. The comparison of the isentropic Mach number \(M_{in}\) is given in Figure 3. It can be seen from the comparison that the numerical result (CFD) is in excellent agreement with experiment (EXP), which demonstrates that the chosen CFD method has high accuracy and reliability, and it is suitable for the calculation and analysis in this paper.

Uncertainty quantification calculation and analysis

Uncertainty quantification mainly includes uncertainty modeling of blade geometric variations, batch numerical simulation (CFD), training of surrogate model and calculation analysis based on surrogate model, which is shown in Figure 4. Through the complete process, the actual uncertainty distributions of turbine blade aerodynamic parameters under the influence of geometric deviation will be obtained.

Uncertainty modeling of blade geometric variations

Since there is a great correlation between the variations of geometry coordinates of different positions on turbine blade surface, the stochastic process model can simulate them well (Schillings, et al. 2011; Papadimitriou, et al. 2016; Luo, et al. 2019). With reference to existing researches, assuming that the blade processing errors obey a Gaussian distribution (Ma
et al. 2017), and the Gaussian stochastic process was used for the uncertainty modeling of blade geometric variations (Razaaly, et al. 2019). According to the processing experience (Editor-in-Chief Committee of Aviation manufacturing engineering handbook, 1997&2016), the average geometric deviation of each point was set as zero, and the surface profile tolerance $c$ was assumed as 0.23mm. Meanwhile, according to the 3-$\sigma$-criterion, the standard deviation of the Gaussian Process was taken as 1/3 of the profile error $c$.

$$m(s) = E[e(s, \varphi)] = 0, \forall s \in \Gamma, \varphi \in \Omega \quad (2)$$

$$\sigma(s) = c(s) / 3 \quad (3)$$

where $s$ and $\varphi$ represent the value of different positions and random cases, respectively, and $\Gamma$ and $\Omega$ represent the blade profile position space and probability space, respectively. Referring to the existing research (Schillings, et al. 2011; Razaaly, et al. 2019), the covariance function of the quadratic exponential attenuation by the surface distance was selected.

$$\text{cov}(s_1, s_2) = \sigma(s_1)\sigma(s_2) \exp\{-k \cdot t^2\} \quad (4)$$

$$t = \frac{l_{ab}}{b} \quad (5)$$

$$l_{ab} = \min\{L_{a-b}, L_{b-a}\} \quad (6)$$

The surface distance $l_{ab}$ is defined as the smaller distance required from point a to point b in the forward and reverse directions on the blade surface, as shown in Equation (6). And the parameter $t$ is the standardized distance parameter by the maximum value of $l_{ab}$. In Equation (4), the parameter $k$ is used to characterize the attenuation rate correlation between the two points. For two points of the same distance, the correlation value will decrease with the increase of $k$ value, that is, the independence will increase and the geometric disturbance waveform will be more disordered, and vice versa. The attenuation rate of partial variation contribution rate of the first 10 principal components under different $k$ values in the principal component analysis (PCA) (Garzon, et al. 2003) is shown in Figure 5.

According to the distribution of geometric variations corresponding to different $k$ values, $k=5$ was selected in this paper. And the corresponding normal coordinate deviation values at different positions on the blade surface are shown in Figure 6, where the point 0 represents the trailing edge point, and the left side and right side represent the pressure side and suction side, respectively. It can be seen that the deviation distributions of the blade surface are smoothly and continuously, and the overall distribution is approximately Gaussian, which basically does not exceed the given tolerance of 0.23mm, proving the effectiveness of this method.

Further, according to the stochastic process model, the covariance matrix of normal coordinate deviation of each point on the blade was obtained, and the PCA method was used. Random deviations were imposed to the nominal blade geometry to obtain random blade geometry samples for the subsequent analysis. The first 7 principal components are considered in the PCA as the corresponding cumulative variance contribution rate is greater than 99%, which can basically fully characterize the variation characteristics of the blade geometry.

![Figure 5 Partial variation contribution rates of PCA modes of stator with different $k$ values](image1)

![Figure 6 Circumferential distribution of normal geometric deviations at mid-span section: $k=5$](image2)

**Training of surrogate model**

The Latin hypercube sampling method (LHS) was applied to take 50 sets of Gaussian random samples as the coefficients of these principal components. The corresponding 50 blade profiles were generated by the above parametric profiling method, and the corresponding 50 sets aerodynamic performance parameter values were obtained through batch numerical simulation calculation. The LHS samples and the corresponding performance values were used as the input and output variables in the training of surrogate model.
The Kriging surrogate model based on polynomial chaos expansions (PC-Kriging) (Kersaudy, et al. 2015) is an effective surrogate model used in this paper. And the leave-one-out cross-validation (LOOCV) error (Blatman, et al. 2010) is used to evaluate the accuracy of the trained surrogate model, and the LOOCV error values of the estimated \( L \) and \( \beta_o \) are \( 6.1 \times 10^{-3} \) and \( 2.4 \times 10^{-3} \), respectively. According to the definition of this error, the error magnitude of these two parameters is obviously less than their standard deviations, meaning that the trained surrogate model meets the requirements of the subsequent uncertainty quantification calculations.

Results of uncertainty quantification of Reference blade profile

10000 sets of random values as the coefficients of principal components were obtained through the ordinary Monte Carlo sampling, and the corresponding \( L \) and \( \beta_o \) are computed by the trained surrogate model. The actual distribution histograms of the two performance parameters are shown in Figure 7, and the performance parameters of the nominal blade profile (expressed as nominal values, the same below) and the corresponding first four statistics are shown in Table 2. The actual \( L \) and \( \beta_o \) are significantly affected by the geometric variations, and the influence magnitude is significantly larger than that of the numerical simulation errors and surrogate model estimation errors. The actual mean value of \( L \) is obviously larger than the nominal value and the relative standard deviation is 10.1%, meaning that there are a clear overall increasing trend and random fluctuations. In addition, according to the skewness and kurtosis, the actual value of \( L \) is likely to be significantly larger than the nominal value, which means a great deterioration of the aerodynamic performance. The standard deviation of \( \beta_o \) is large, and the actual value is widely dispersed near the nominal value. According to the above analysis, the influence of geometric variations is greatly significant, and it is necessary to comprehensively consider this influence in the design process.

![Figure 7 Histograms of aerodynamic parameters](image)

(a) \( L \)

(b) \( \beta_o \)

Figure 7 Histograms of aerodynamic parameters (a) \( L \); (b) \( \beta_o \)

| Statistics of \( L \) and \( \beta_o \) of the Reference blade profile |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Nominal Mean | Std. Dev. | Skewness | Kurtosis |
| \( L \) | 0.0397 | 0.0040 | 0.79 | 13.21 |
| \( \beta_o \) | 59.09 | 0.35 | -0.40 | 3.87 |

SENSITIVITY ANALYSIS OF DESIGN PARAMETERS

Design of experiment

In this paper, the influence of different leading and trailing edge design geometry on the uncertainty performance of the turbine blade under geometric variations was mainly analysed. The 5 design parameters of interest included short/long radius of leading-edge ellipse \( r_1/r_1 \), radius of trailing edge \( r_2 \) and leading/trailing edge wedge angles \( \omega_1/\omega_2 \). And the rest 7 parameters of all the 12 parameters were fixed as the same as those of the Reference blade profile. Based on actual design experience, the distribution types and parameters of these 5 parameters are shown in Table 3, in which \( r_1 \) and \( r_2 \) are standardized by the \( b \) as \( s_{r1} \) and \( s_{r2} \), respectively, and \( r_{12} \) is replaced by the long/short radius ratio \( r_{12} \). 50 sets of parameters were selected by the LHS sampling method, and the generated blade profiles are as shown in Figure 6. It can be seen that the leading-edge/trailing-edge geometry can be effectively adjusted by the profiling method, which can meet the needs of the subsequent analysis. Therefore, the effectiveness of the profiling method is fully proved.

![Table 3 Distribution type and parameters of design parameters](image)

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Distribution type</th>
<th>Distribution parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{r1} = r_{11}/b_x )</td>
<td>Uniform</td>
<td>([1.0, 3.0])</td>
</tr>
<tr>
<td>( r_{12} = r_{11}/r_{12} )</td>
<td>Uniform</td>
<td>([1.0, 4.0])</td>
</tr>
<tr>
<td>( s_{r2} = r_{2}/b_x )</td>
<td>Uniform</td>
<td>([0.6, 1.2])</td>
</tr>
</tbody>
</table>
For the generated 50 blade profiles, each blade profile was used as the current nominal geometry for uncertainty quantification under the influence of geometry variations. The methods and parameters in each step of the uncertainty quantification were the same as those in the above description. Through uncertainty quantification, the actual aerodynamic performance parameter values under the influence of geometry variations were obtained for the subsequent analysis.

Analysis of uncertainty quantification results
Based on the above-mentioned uncertainty quantification method, the uncertainty analysis of the influence of geometric variations on the generated 50 blade profiles was carried out. The nominal values, and mean values and standard variations of the $L$ and $\beta_0$ of these blade profiles are shown in Figure 9 and Table 4.

<table>
<thead>
<tr>
<th>Performance parameters</th>
<th>$L$ (Nominal)</th>
<th>$L$ (Mean)</th>
<th>$L$ (Std. Dev.)</th>
<th>$\beta_0$ (Nominal)</th>
<th>$\beta_0$ (Mean)</th>
<th>$\beta_0$ (Std. Dev.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference profile</td>
<td>0.0397</td>
<td>0.0409</td>
<td>0.0040</td>
<td>59.09</td>
<td>59.02</td>
<td>0.35</td>
</tr>
<tr>
<td>Min.</td>
<td>0.0376</td>
<td>0.0380</td>
<td>0.0007</td>
<td>58.19</td>
<td>58.15</td>
<td>0.22</td>
</tr>
<tr>
<td>Max.</td>
<td>0.0533</td>
<td>0.0577</td>
<td>0.0180</td>
<td>59.52</td>
<td>59.52</td>
<td>0.65</td>
</tr>
</tbody>
</table>

According to the results, it can be found that the difference between the nominal performance values and the actual dispersed performance values is similar to that of the Reference profile. That is, the mean values are basically consistent with the nominal values, which has some deterioration, and the actual performance has a certain degree of dispersion. There is a clear positive correlation among the nominal values, actual mean values, and the actual standard deviation values of $L$. This means that the poor design of the leading-edge/trailing-edge geometry will make these performance values deteriorate at the same time. On the other hand, the better design of the leading-edge/trailing-edge geometry can effectively improve the two performance values at the same time and enhance the overall aerodynamic performance.

Sensitivity analysis
In this paper, Sobol sensitivity analysis (Sobol, et al. 1993) and Spearman rank correlation (SRC) (Iooss, et al. 2015) were used to analyse the influence degree of the 5 design parameters on the actual $L$ and $\beta_0$ affected by the geometric variations. The Sobol method is a global sensitivity analysis method, which decomposes the variance of the dependent variables into fractions attributed to each independent variable in the global scope and the sensitivity is based on the relative variance contribution magnitude. In the SRC method, the direction and degree of rank correlation between different variables are computed as the direction and degree of the sensitivity. The samples needed in SRC are less than other sensitivity methods, while the analysis result are stable and reliable, which is suitable for the sensitivity calculation in this paper (Lange, et al.2010). The obtained sensitivity on the mean values and standard variations of $L$ and $\beta_0$ is shown in Figure 10 and Figure 11.
According to the results of sensitivity analysis, the $r_{11}$ has the most significant impact on the nominal value, mean value and standard deviation of $L$ and $\beta_0$, which is shown a relatively obvious negative correlation. An appropriate increase of $r_{11}$ will make the nominal value, mean value and standard deviation of loss effectively reduced, and the actual aerodynamic performance improved. The effect of long/short radius ratio $r_{12}$ and leading wedge angle $\omega_1$ is slightly smaller, and the influence of the trailing-edge geometry is significantly smaller than that of the leading-edge region. The 50 sets of calculation results sorted by is $sr_{11}$ shown in Figure 12, and it can be seen that the increase of $sr_{11}$ can effectively reduce the mean value and standard variation of $L$, and as $sr_{11}$ exceeds 2.0%, both the mean value and standard variation are at a relatively low level. This is consistent with the results of the above sensitivity analysis.
MECHANISMS ANALYSIS OF EFFECT OF LEADING-EDGE DESIGN PARAMETERS

The influence mechanisms of the leading-edge geometry on the actual aerodynamic performance were analysed in detail in this chapter. The parameters $r_{11}$, $r_{12}$ and $\omega_1$ were selected as the research objects, and were analysed separately, while the other geometry parameters were kept as the same as those of the Reference blade profile.

Effect analysis of short radius ratio of leading-edge ellipse

The uncertainty calculation results with different values of $sr_{11}$ are shown in Table 5. With $sr_{11}$ increasing from 1.00% to 2.30%, the nominal value and mean value of $L$ are significantly reduced, and the difference between the two values is gradually decreasing and tends to disappear. And the change of the parameter $\beta_o$ of also shows a great improvement of the nominal and mean performance. At the same time, the standard deviation of both $L$ and $\beta_o$ are also gradually decreasing, that is, the robustness of the actual aerodynamic performance is significantly improved.

<table>
<thead>
<tr>
<th>$sr_{11}$ (%)</th>
<th>$L$</th>
<th>$\beta_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0499</td>
<td>58.47</td>
</tr>
<tr>
<td>1.65</td>
<td>0.0392</td>
<td>59.10</td>
</tr>
<tr>
<td>2.30</td>
<td>0.0380</td>
<td>59.20</td>
</tr>
</tbody>
</table>

In order to analyse the total pressure loss in detail, the total pressure loss $L$ was divided into three parts: the boundary layer loss in the pressure side $L_p$, the boundary layer loss in the suction side $L_s$, and the wake mixing loss $L_w$. A general cumulative total pressure loss coefficient $L_c$ was used to describe the cumulative total pressure loss on the blade surface, which was defined by Wang (2019). The $L_p$ and $L_s$ are the values of the $L_c$ at the end of the pressure side and the suction side respectively, and the wake mixing loss $L_w$ is the remaining item in $L$.

This decomposition method was applied and the obtained statistics of the sub-region loss with different values of $sr_{11}$ are shown in Table 6. The Pearson correlation coefficient represent the linear correlation and is used for the variance summation formula, and the Spearman correlation coefficient represent the global monotonicity between the two variables. It can be seen that the total loss $L$ and its variations mainly come from $L_p$ and $L_w$, while $L_p$ is relatively small and basically unchanged, which will be neglected. With $sr_{11}$ increasing from 1.00% to 2.30%, the mean value and standard deviation of $L_p$ and $L_w$ are significantly reduced. On the other hand, with the increase of $sr_{11}$, the correlation between $L_p$ and $L_w$ are changed from positive to negative, which is also shown in Table 6. The decrease of the standard deviation of $L_p$ and $L_w$ and the change of the correlation value are the main reason for the decrease of the standard deviation of $L$.

<table>
<thead>
<tr>
<th>$sr_{11}$ (%)</th>
<th>Mean of sub-region loss</th>
<th>Std. Dev. of sub-region loss</th>
<th>$L_p - L_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0013</td>
<td>0.0246</td>
<td>0.0271</td>
</tr>
<tr>
<td>1.65</td>
<td>0.0011</td>
<td>0.0211</td>
<td>0.0188</td>
</tr>
<tr>
<td>2.30</td>
<td>0.0011</td>
<td>0.0201</td>
<td>0.0171</td>
</tr>
</tbody>
</table>

Figure 13 Axial distribution of $L_p$ on suction side with different values of $sr_{11}$

The axial distribution of $L_p$ on the suction side with different values of $sr_{11}$ is shown in Figure 13. It can be seen that the distribution of $L_p$ is basically similar, and the separation bubble starts to appear at 75%-80% axial position, causing obvious increase and dispersion of the loss in the rear region. With $sr_{11}$ of 2.30%, the distribution with different blade geometries is basically the same, the overall loss is relatively small, and there are some differences only in the region after the separation. With $sr_{11}$ of 1.00%, the overall loss is relatively large, and there is obvious dispersion. The loss difference between these cases is mainly due to the difference in the leading-edge region before the 10% axial position, while the increase of boundary layer loss at the 10%-75% axial position is basically the same. In addition, in the case of $sr_{11}$ 1.00%,
the separation loss is also generally slightly larger than that of \( sr_{11} \), 2.30\%, and obvious open separation may occur, which will lead to obvious aerodynamic loss. The case of \( sr_{11} \) 1.65\% is between the cases of \( sr_{11} \) 2.30\% and radius \( sr_{11} \) 1.00\%.

Due to the high load of the studied blade profile, the \( L_e \) is relatively large. Affected by manufacturing geometric variations, there will be a certain variation in leading-edge geometry, causing the variation in the actual leading-edge metal angle, that is, variation in the incoming attack angle. When the leading-edge radius is small (the case of \( sr_{11} \) 1.00\%), the flow conditions and loss levels near the leading edge are more sensitive to the incoming attack angles. The actual large positive attack angle will lead to obvious separation bubble at the suction side of leading edge, and great aerodynamic loss near leading edge. At the same time, the uncertainty in the incoming attack angle will cause obvious uncertainty in this aerodynamic loss, which is shown in Figure 10(a). When the leading-edge radius is large (the case of \( sr_{11} \) 2.30), the sensitivity of the flow conditions and loss levels near the leading edge to the incoming attack angles will be significantly reduced, so the degree of the loss and its uncertainty in this region will be significantly reduced, which is shown in Figure 10(c).

The wake mixing loss of turbine blade mainly depends on the thickness of the boundary layers on both sides near the trailing edge, and the thickness of the trailing edge (Denton, 1993). Based on this, the correlation between suction surface loss \( L_s \), wake loss \( L_w \) and trailing edge thickness \( w_t \) with different values of \( r_{12} \) was calculated, and the results are shown in Table 7. With \( sr_{11} \) of 2.30\%, there is an obvious negative correlation between \( L_s \) and \( L_w \), mainly because they have obvious negative correlation and positive correlation with \( w_t \) respectively. The increase of trailing-edge thickness will directly lead to the increase of wake mixing loss, while weaken the back-pressure degree of the rear section of suction side and reduce the separation loss at the same time. Thus, the two main losses are negatively correlated, which is conducive to the reduction of the standard deviation of \( L \). With \( sr_{11} \) of 1.00\%, the uncertainty of \( L \) mainly comes from the uncertainty of the loss near leading edge, that is, from the geometric variations of leading edge. As for \( L_w \), the suction surface loss near trailing edge is great and the boundary layer is thick, and as \( L \) increases, more low-energy fluid flows into the mixing area, and the \( L_s \) increases as a result. The influence of \( w_t \) on \( L_e \) and \( L_w \) is relatively small. Therefore, there is an obvious positive correlation between \( L_e \) and \( L_w \), and the standard deviation of the total loss is relatively large. The correlation between these in the case of \( sr_{11} \) 1.65\% is between the cases of \( sr_{11} \) 2.30\% and \( sr_{11} \) 1.00\%.

<table>
<thead>
<tr>
<th>( sr_{11} ) (%)</th>
<th>Spearman correlation</th>
<th>Pearson correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.90 -0.12 -0.02</td>
<td>0.97 -0.26 -0.19</td>
</tr>
<tr>
<td>1.00*</td>
<td>0.89 -0.01 0.04</td>
<td>0.89 -0.06 0.00</td>
</tr>
<tr>
<td>1.65</td>
<td>0.01 -0.47 0.63</td>
<td>0.66 -0.41 0.24</td>
</tr>
<tr>
<td>2.30</td>
<td>-0.71 -0.65 0.97</td>
<td>-0.71 -0.70 0.96</td>
</tr>
</tbody>
</table>

1.00* means that the case with an obvious open separation is eliminated in the correlation calculation.

Effect analysis of long/short radius ratio of leading-edge ellipse

The uncertainty calculation results with different values of the long/short radius ratio \( r_{12} \) are shown in Table 8. It can be seen that the influence of \( r_{12} \) is smaller than that of \( sr_{11} \). Similar to the case of \( sr_{11} \) 2.30\%, the leading-edge is relative thicker in the case of \( r_{12} \) 1.50, and the actual uncertainty performance is relative better. With \( r_{12} \) increasing from 1.50 to 3.00, the nominal value, mean value and standard deviation of \( L \) will be gradually increased, and the actual aerodynamic performance under the influence of geometric variations will deteriorate.

<table>
<thead>
<tr>
<th>( r_{12} )</th>
<th>( L )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal</td>
<td>Mean</td>
</tr>
<tr>
<td>1.50</td>
<td>0.0395</td>
<td>0.0407</td>
</tr>
<tr>
<td>2.25</td>
<td>0.0392</td>
<td>0.0410</td>
</tr>
<tr>
<td>3.00</td>
<td>0.0399</td>
<td>0.0429</td>
</tr>
</tbody>
</table>

Effect analysis of leading-edge wedge angle

The uncertainty calculation results with different values of \( \omega_1 \) are shown in Table 9. With \( \omega_1 \) of 25.0\°, the nominal values, mean values and standard deviations of \( L \) and \( \beta \) are clearly better than those of \( \omega_1 \) of 15.0\°/20.0\°. That is, the actual aerodynamic performance under the influence of the geometric variations will be improved with the increase of \( \omega_1 \). The influence degree of \( \omega_1 \) is between that of \( r_{11} \) and \( r_{12} \).
The axial distribution of $L_c$ on the suction side with different values of $\omega_1$ is shown in Figure 14, and the correlation coefficients between $L_c$, $L_w$ and $w_s$ are shown in Table 10. Similar to that of the $\omega_{11}$, the loss difference between these cases is mainly in the leading-edge region before the 10% axial position, and the loss in the rest region is basically the same. The value of $\omega_1$ can give a rough representation of the magnitude of discontinuity in curvature in the connection area (Benner, et al. 1997), and a larger value means a better continuity. With $\omega_1$ of 15.0°, the discontinuity in curvature is great, which will deteriorate the flow state and promote its sensitivity to inflow attack angle in the leading-edge region. And the loss in this region is great and dispersed, and this deterioration will spread to the entire boundary layer of suction side. As a result, the mean value and standard variation of $L_s$ are both large. At the same time, the great loss will affect the wake mixing, and similar to the case of $\omega_{11}$ 1.00%, the mean value and standard variation of $L_w$ are large, and there is a great positive correlation between $L_w$ and $L_m$, making the total loss $L$ great and scattered. The increase in $\omega_1$ will promote the continuity and improve the flow state in the leading-edge region, and the overall aerodynamic performance is better.

![Figure 14 Axial distribution of $L_c$ on suction side with different values of $\omega_1$](image)

<table>
<thead>
<tr>
<th>$\omega_1$ (°)</th>
<th>Mean of sub-region loss</th>
<th>Std. Dev. of sub-region loss</th>
<th>$L_s - L_w$</th>
<th>Spearman</th>
<th>Pearson</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.0</td>
<td>0.0011</td>
<td>0.0222</td>
<td>0.0207</td>
<td>0.0003</td>
<td>0.0025</td>
</tr>
<tr>
<td>20.0</td>
<td>0.0011</td>
<td>0.0211</td>
<td>0.0188</td>
<td>0.0003</td>
<td>0.0015</td>
</tr>
<tr>
<td>25.0</td>
<td>0.0011</td>
<td>0.0204</td>
<td>0.0175</td>
<td>0.0002</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

In the actual design of turbine blade under the influence of the processing geometric variations, the uncertainty performance parameters such as the mean value and standard deviation of the aerodynamic performance should be comprehensively considered. According to the above analysis results, in the uncertainty design of 2D turbine blades, after the key geometry parameters such as stagger angle $\beta$ and pitch $t$ are given, the leading-edge geometry, especially the leading-edge thickness, should be paid attention to. It is recommended to select a relatively thick leading-edge geometry in the design of high-loading low-pressure turbine blade to reduce the influence of geometric variations.

CONCLUSIONS

Through the research, the following conclusions can be drawn,

1. The parametric blade profiling method used in this paper is effective for the profiling of leading-edge/trailing-edge geometry, and is also convenient for the analysis. The uncertainty quantification method used in this paper can be used for the effective analysis for the influence of processing geometric variations on the aerodynamic performance of turbine blade.

2. The geometric variations presented in this paper have a significant impact on the actual aerodynamic performance of turbine blade. Under the uncertainty effect, compared with the nominal value, the mean value of total pressure loss increases by 7.9%, the relative standard deviation is 10.1%, and the standard deviation of outflow angle is 0.35°. In addition, there is a great possibility that the actual performance will deviate significantly from the nominal value, resulting in serious impact. Enough attention should be paid to the actual uncertainty influence caused by the processing geometric variations.

3. For the high-loading low-pressure turbine blade in this paper, the leading-edge geometry has a significant influence on the actual uncertainty aerodynamic performance, which is greater than that of the trailing-edge geometry. The flow near the leading-edge has a significant impact on the whole suction-side boundary layer flow and wake mixing. The increase of leading-edge thickness and wedge angle can restrain the sensitivity of leading-edge geometric variations, improve the stability of leading-edge flow state, and reduce the uncertainty influence on the whole aerodynamic performance. In practical design, large leading-edge thickness and wedge angle can be selected appropriately to improve the robustness of actual aerodynamic performance.

NOMENCLATURE

- $b$ maximum value of surface distance $l$ (m)
- $c$ surface profile tolerance (m)
**References**


