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SINGULAR SPECTRUM ANALYSIS OF TRANSIENT HEAT RELEASE IN A PREMIXED SWIRL COMBUSTOR

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ABSTRACT

Lean premixed swirling flame is normally accompanied with a thermoacoustic issue. Analysis of the heat release rate time series generated by large eddy simulation is an effective way to determine the flame dynamics during combustor design. However, due to the high cost of large eddy simulation, the signal is always short-duration. It is difficult to tell if the flames would eventually quench from such duration signals. Therefore, there should be one post-processing method, which can analyze such non-stationary, short, and noisy time series and extract useful flame dynamic information. In this paper, singular spectrum analysis was performed to do such job. Flame transfer path response is defined to identify the flame response of each singular spectrum mode. The mode shapes extracted from dynamic mode decomposition were compared with transfer path response results. Results indicate that traditional flame transfer functions show higher gain with a wide frequency range and critical eigenfrequencies are masked. Via singular spectrum analysis, the reconstructed series can track the original heat release time series. Distinct evolution trends and frequency responses can be found in those modes. The transfer path response results of single or grouped singular spectrum modes may reflect the heat release rate response to excitation more accurately than with traditional analysis. The frequency bands of singular spectrum modes correspond to those of dynamic mode decomposition. Compared with decomposed modes, frequency responses may be caused by shear layer instabilities and flame motions.

INTRODUCTION

When reducing pollution for gas turbine or aero-engine, one technology is using lean premixed flame. When the phase between the heat release and pressure wave is close and their product is larger than the system damping, unstable combustion becomes an issue (Ying and Vigor, 2009). Oscillation of the heat release rate (HRR) is the parameter to account flame behaviour. Analysing the time series of HRR is the centre point for current paper. Assuming the combustion system is a linear black box, then its behaviour can be extracted by using impulse response to get the flame transfer function (FTF) (Polifke, 2020). By obtaining open-loop FTF, it is much easy for hardware down selection (Yang et al., 2020). Considering wrinkled flame fronts, which are caused by coherent structures. The numerical method, large eddy simulation (LES), which resolves such structures, is an approved methodology to extract FTF (Yang and Ker, 2012; Luo et al., 2020). This provides to possible to investigate the source for high gain in FTF. Especially useful for the frequency bands which are influenced by flame structures (Pritz et al., 2010). Nevertheless, due to the high computational cost of LES, the extracted information might not enough in statistical means. The short-duration time series of HRR might be not obvious to give the conclusion that the signal is stationary but still be used to perform FTF, which leads to high gain in the low frequency bands. Though the signal could not give proper low frequency results, still owns medium and high frequency information inside. Thus, it would be useful if there is an algorithm can decompose signals into very low frequency part and the result part. It helps to identify the quenching trend, and also helps to gain potential information from signal without wasting. Therefore, the stationary time series of HRR is decomposed using one data-driven algorithm, Singular Spectrum Analysis (SSA). The SSA method was first proposed by Colebrook to describe the geographical change (Colebrook, 1978) and soon been used to studied nonlinear dynamic system (Broomhead and King, 1986) and showed its potential dealing with noisy, short duration chaotic system (Vautard et al., 1992). Therefore, the thermoacoustic instability in a self-excited tube combustor
was firstly reported in (Noble et al., 2009). Though SSA proves its capable of extracting nonlinear characteristics (Elsner and Tsonis, 2013), such method cannot indicate the corresponding flow structures those aerodynamic designers would use to optimize the design (Buschhagen et al., 2019). To link the frequency response to flame dynamics and the flame structures, one swirl-stabilised partially premixed flame is studied in current paper. In such flame, the time series of HRR is stationary and indicates the flame is stable. Since the time series has only one variable, the trajectory matrix of the series is used as the first step of SSA. The SSA method further decomposes the time series of HRR into narrow frequency bands. By analysing the stand-alone SSA modes and their response to inlet excitation, the stability of each mode can be estimated. The later part is defined as a new parameter called transfer path response (TPR). The frequency spectrum of HRR based on FFT was compared with TPR. Finally the dynamic mode decomposition (DMD) (Schmid, 2010; Rowley et al., 2009) analysis, which is normally used to predict evolution of the mode, such as growth, decay, and oscillation (Palies et al., 2017), is combined with SSA to explore the relationship between combustion instabilities and flame-vortex interactions (Huang et al., 2016).

**METHODOLOGY**

**CFD setup for one swirl stabilized flame**

The simulation domain of the LES is 1/8 of the full geometry as shown in Figure 1 (left). Swirl flow is generated by an axial swirler with nozzles on the blades. As shown in Table 1, the geometrical parameters are, the mixing zone length is \( l \). The radius of centre lance \( R_t \) is used as characteristic length. The radius of the end of centre lance is \( R_{1,TE} \), and the diffusor angle \( \alpha \). With proper designed swirl number, the streamwise vorticity is generated (Quinlan and Zinn, 2017) and finally causes the recirculation zone. Table 2 lists the boundary conditions. The equivalence ratio is 0.514. This is atmospheric condition.

![Table 1](image)

**Table 1 Geometric parameters (normalised by the radius of centre lance R1)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>( \alpha ) (°)</th>
<th>( R_d/R_1 )</th>
<th>( R_{1,TE}/R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{d}{2\times R_1} )</td>
<td>0.5</td>
<td>15</td>
<td>6.25</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 1 Schematic of the Geometric Parameters (left) and LES Mesh (right)**

![Figure 1](image)

**Table 2 Inlet Boundary Conditions for LES Calculation**

<table>
<thead>
<tr>
<th>Items</th>
<th>Velocity (m ( \cdot ) s(^{-1} ))</th>
<th>Temperature (K)</th>
<th>Species</th>
<th>Turbulence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow air</td>
<td>50.0</td>
<td>600</td>
<td>21% vol. O(_2)</td>
<td>low</td>
</tr>
<tr>
<td>Fuel</td>
<td>85.0</td>
<td>300</td>
<td>Pure methane</td>
<td>high</td>
</tr>
</tbody>
</table>

LES was enabled from converged reactive steady RANS. The Favre-filtered transport equations for mass and momentum are shown in Eq. (2) and (3).

\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j) = 0
\]

(1)

\[
\frac{\partial (\bar{\rho} \bar{u}_j)}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_i \bar{u}_j) = \frac{\partial}{\partial x_i} \left[ \bar{\rho} \left( \bar{u}_i \frac{\partial \bar{u}_j}{\partial x_i} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial \bar{u}_k}{\partial x_k} \right) \right] + \bar{\rho} \bar{u}_i \bar{u}_j - \frac{\partial \bar{p}}{\partial x_j}
\]

(2)

Steady RANS has been used to estimate first LES mesh (Pope, 2004). Then the mesh is refined based on Celik-defined energy-based index \( M \) with more than 85% (Pope, 2004; Celik et al., 2009). Polyhedral cells with three-layer prism mesh were generated. The mixing zone and flame region has been refined as presented in Figure 1(right) and corresponding mesh resolution is less than 0.5mm. The total amount of cells is 4.7 million.
In LES, the Favre-filtered transport equations are applied for momentum and mass conservation. Here used wall-adapting local eddy-viscosity model. The second-order implicit scheme is applied for transient formulation. The bound centre difference scheme is applied for spatial discretization. There is no turbulence perturbations are set for inlets in order to maintain the excitation signal shape. The transport equation of the progress variable (c) is set for combustion simulation.

The source term in the turbulent flame closure (TFC) (Zimont et al., 1998) model is closed by solving the turbulent flame speed. The GRI 3.0 mechanism is used for the tabulation (http://www.me.berkeley.edu/gri_mech/). Heat loss is not considered in the tabulation or simulation. The combustion model and LES strategies are evaluated also in a V-type bluff body stabilized premixed combustion and compared with experimental data. The atmospheric test for this swirler are ongoing. The performance tests and FTF tests are scheduled and preliminary results indicate the same flame shape. It should also be mentioned that the proposed post-processing method is an prediction method, which needs to be perform before test.

Flame transfer function (FTF) and transfer path response (TPR)

In the linear time-invariant (LTI) system, the FTF is defined by Eq. (1) in frequency domain.

$$\text{FTF}(\omega) = G(\omega) \exp(i\phi(\omega)) = \frac{\hat{Q}}{\hat{u}}$$  \hspace{1cm} (3)

Here, the HRR time series (Q̃) is extracted from LES. ù is the inlet air velocity perturbation, ω is the complex angular frequency. G(ω) and φ(ω) are the gain and phase. The excited LES lasts 0.25 s. The algorithm is the same as the general frequency response function algorithm (Isermann and Munchhof, 2010; Huber and Polifke, 2009). The excitation signal uses discrete random binary signal (DRBS) (Isermann and Munchhof, 2010) with 5% amplitude and in the frequency less than 800 Hz. This signal has already been successfully applied to a lab-scale and and industrial combustor (Bothien et al., 2019; Yang et al., 2015; Hubschmid et al., 2014). Transfer path response means the transfer function between the excitation and decomposed HRR. The formulation is similar, while the Q̃ is substituted by decomposed SSA modes.

**Singualar spectrum analysis (SSA)**

In the SSA, assume the window is L, the SSA algorithm is the following steps:

1. Embedding. The subset Yt can form the trajectory matrix with the window length L implemented. Larger L leads to more singular variables thus loss information. The window length should not be too small which causes the RAM cost and computation duration. Here, the L < n/2 law is applied.

2. Using singular value decomposition (SVD) algorithm \( Y = U \Sigma V^T \). Here \( U_{L \times L}, V_{(n-L+1) \times (n-L+1)} \) are the left and right singular vectors of \( Y \), respectively. The diagonal matrix \( \Sigma_{L \times L} \) contains eigenvalue \( \lambda_i \).

3. Reconstruction. \( a^m \), which represents the weight of temporal evolution, is calculated as \( Y \hat{u}_m \). The matrix composed by \( a^m \) is right vector \( \sqrt{\lambda} V^T \). Then the reconstruction is achieved by averaging anti-diagonals of the elements.

4. Selection. Since the eigenvalues from SVD are already ordered in a decreasing fashion, the first k modes are selected for discussion. The \( R^2 \) score and mean squared error (MSE) are used to evaluate, though not presented here.

Here, separate SSA modes with same frequency bands are not grouped to prevent phase influence.

**Dynamic mode decomposition (DMD)**

The spatial-temporal information is extracted from LES with constant interval. Concatenate vectors and form the data matrix into \( X_t \) \( (t = 0, 1, ..., n-1) \) and \( Y_t \) \( (t = 1, ..., n) \), where t is the time. Since the time step is small enough, there exists linear operator M, \( Y = MX \). Then one can deduces \( MXX^{-1} = YY^{-1} \), which leads \( AI = M'M^{-1}, X = U\Sigma V^T \). The left-singular vectors \( U \in (m \times m) \) and the right-singular vectors \( V^T \in (n \times n) \) are calculated. Project M on matrix U, which leads to a low-rank \( M = U'MU \). Then \( M = U'V\Sigma_i^{-1} \). Then one can do eigen decompose matrix on \( M \) as \( \tilde{M}W = \Lambda W \), in which \( W \in (r \times r) \) is the singular vector diagonal matrix, and r is the rank. The dynamic mode is calculated as \( \Phi = X'V\Sigma_i^{-1}W \). The growth rate is calculated as \( a = Re(\log\lambda_i)/(2\pi \cdot dt) \). The growth rate means the fluctuation evolution with time. A positive value indicates the fluctuation increases with time. The frequency is calculated as \( f = Im(\log\lambda_i)/(2\pi \cdot dt) \). Re, Im represents the real part imaginary part, respectively.

The detail algorithm of DMD has been introduced in previous work (Yang et al., 2020).

**RESULTS**

**Flame characteristics and FTF**

The ensembled averaged PV and root-mean-square (RMS) of PV contour plots in the centre plane are shown in Figure 2 (left and right, respectively). Results indicate that there is flame forming (PV ≈ 1) in the centre recirculation zone. The RMS contour indicates that there is stronger mixing for the inner flame branch than the outer one.

Figure 2 Contour of Ensembled Averaged PV (left) and RMS PV (right) in the Centre Plane

Figure 3 Time series of Volume Integral Heat Release Rates and Corresponding Flame Transfer functions

The $Q$ series is collected during simulation with a sampling frequency of 10,000 Hz. As shown in Figure 3a, HRR shows stationary, which is proved using statistical test. The FTF is shown in Figure 3b, and the gain in the frequency range less than 600Hz is all less than 1. The FTF indicates stable flame.

SSA results

The TPR is the response of each HRR-SSA mode to the inlet excitation. Although the complex eigenvalues come in pairs for the trajectory matrix, the single mode is calculated based on the sum of the products of the left and right eigenvectors. As shown in Figs. 4-8, the SSA modes and their spectrum (red) are compared with the TPR results illustrated by gain and phase plots (blue). Corresponding modes include their PSD as 400Hz, 300Hz, 50Hz, and 200~220Hz. Comparing to the FTF result, there is no similar frequency bands. On the contrary, the frequency bands of TPR show similar frequency bands as shown in the FTF results. The first two SSA modes correspond to 200Hz. The 3rd and 4th SSA modes correspond to 18 Hz. The 6th and 7th SSA modes correspond to 110Hz. And the 9th and 10th SSA modes correspond to 110Hz and 180Hz. Those SSA modes are not perfect harmonic, which is because that the SSA algorithm is not FFT.

Figure 4 SSA Mode 1st (Same for the 2nd Mode): Time Series and It’s Spectrum (red) and TPR Gain and Phase (blue)
Figure 5 SSA Mode 4th (Same for the 3rd Mode): Time Series and It’s Spectrum (red) and TPR Gain and Phase (blue)

Figure 6 SSA Mode 5th (Same for the 8th Mode): Time Series and It’s Spectrum (red) and TPR Gain and Phase (blue)

Figure 7 SSA Mode 6th (Same for the 7th Mode): Time Series and It’s Spectrum (red) and TPR Gain and Phase (blue)
DMD results

Though the SSA response to FTF results with different distinguishable frequency bands, not all the bands are due to flame-flow interaction. In order to identify the modes due to hydraulic instability, DMD is used to decompose flame field. The first 50 modes are recorded but only those modes with high growth rates are studied.

Compared with the grouped SSA-TPR frequency range in the above table, the frequency bands from DMD and SSA-TPR methods are not the same. Especially, two DMD modes (not presented here) having zero-value frequency. Those modes correspond to mean and RMS shape. There is similar modes in the SSA-TPR results, which implies very low-frequency flame motion.

Table 3 Comparison of Frequency Bands between SSA-PSD, SSA-TPR, and DMD Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>PSD of SSA mode [Hz]</th>
<th>TPR of SSA mode [Hz]</th>
<th>DMD Mode</th>
<th>Frequency [Hz]</th>
<th>Growth rate [1/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1,2</td>
<td>200</td>
<td>19</td>
<td>191</td>
<td>-296</td>
</tr>
<tr>
<td></td>
<td>3,4</td>
<td>300</td>
<td></td>
<td>1,18</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5,8</td>
<td>40</td>
<td>21</td>
<td>93</td>
<td>-3-280</td>
</tr>
<tr>
<td></td>
<td>6,7</td>
<td>200-220</td>
<td></td>
<td>110</td>
<td>-326</td>
</tr>
<tr>
<td></td>
<td>9,10</td>
<td>200-220</td>
<td></td>
<td>110-170</td>
<td></td>
</tr>
</tbody>
</table>
The SSA TPR 200 Hz band corresponds to the 19th DMD mode, which represents the inner and outer flame branches are flapping. If in the further development of such burner shows 200Hz thermoacoustic gain higher than 1, it might be modified by modifying such flapping flame branches. Changing the diffusor angle and burner exit would modify such motion. The 21st DMD mode indicates the shedding flame tip. The corresponding frequency band is approximate 100 Hz. The follow-up development is actually the same as the above analysis.

![DMD Mode Shapes of Heat Release Rate Data on the Snapshots (G.R. is the Abbreviation of Growth Rate)](image)

CONCLUSIONS

This paper studies the time series of heat release rate by using data-driven method singular spectrum analysis. Results indicate that, conventional spectrum analysis cannot provide directly link with flame transfer function since the PSD indicates the self-excited mode, which might not capture by flame. The FTF is rather the forced excited method. Thus the transfer path analysis between each SSA mode and inlet excitation is performed in order to decompose time series of HRR into modes those can be linked with FTF. Although DMD separates more frequency bands than SSA, major eigenfrequencies maintain consistency. Compared with DMD modes, frequency responses in TPR results may be caused by the flapping and shedding motions of the two flame branches.

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