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# IN SITU IDENTIFICATION OF THERMOACOUSTIC STABILITY IN ANNULAR COMBUSTORS

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## ABSTRACT

In annular combustion systems, thermoacoustic eigenmodes can manifest as standing waves, traveling waves or some form in between. Which dynamic solution appears in a combustor depends on details, regarding the flow field and (unintentional) breaking of the cylindrical symmetry of the annular combustion system. When these details are unknown, the specific behavior cannot be predicted from the characteristics of a single burner. Due to the (nearly) degenerate nature of the acoustic solution, annular eigenmodes come in pairs with practically the same eigenfrequency. In order to identify the thermoacoustic modes, conventional analysis of a spectral peak from a measurement does not suffice, because the peak is a superposition of the two eigenmodes. A method has been proposed to identify the two eigenmodes of given azimuthal mode order from multiple simultaneous measurements around the circumference of the combustion system. Using output-only identification on the acoustic signals, it is possible to estimate the individual mode shapes, frequencies and growth rates of the co-existing eigenmode pair. In this work, the strategy is applied to experimental data from an annular combustor. A split in the growth rate pair is observed during stable operation, depending on the equivalence ratio and flame-to-flame distance. It shows that in situ identification of annular thermoacoustics can reveal subtle dynamic effects, which is useful for testing and online monitoring of annular combustors. The point where - and in what form - instability occurs can be foreseen in detail under the current operating conditions.

## INTRODUCTION

In combustion systems with an annular combustion chamber, the low frequency azimuthal modes are a major concern regarding thermoacoustic stability. These modes, in which the flames of individual burners couple with acoustic waves traveling around the combustion chamber circumference, can reach such high amplitudes that a limited operability window must be accepted or structural damage might occur.

Compared to axial thermoacoustic modes, azimuthal modes are more complex. Due to the cylindrical symmetry of the geometry, acoustic solutions consist of pairs with the same wavelength. These degenerate pairs cannot be isolated in Fourier domain and are likely to be coupled as their eigenfrequencies are identical. Existing methods to identify modal damping are not suited for the (nearly) degenerate thermoacoustic mode pairs found in annular combustors.

Another complication is that the flames are excited by transverse acoustics leading to several sources of heat release fluctuations. Following the review paper of O'Connor et al. [1] concerning transverse flame excitation, heat release fluctuations are predominantly generated by axial fluctuations, caused by 'injector coupling'.

Nevertheless, azimuthal fluctuations also influence the heat release directly, for example investigated by Saurabh and Paschereit [2]. These minor contributions to the stability can be decisive on the modal behaviour in both the stable and limit cycle regime. With a model including a nonlinear dependency on azimuthal particle velocity, Ghirardo and Juniper [3] were able to obtain standing thermoacoustic limit cycle solutions. A classical stability analysis approach in which axial Flame Transfer Functions (FTF's) are joined to an annular acoustic network is not able to account for a direct transverse flame response.

Several publications on the dynamic behaviour of azimuthal modes in limit cycle have been published, including Worth and Dawson [4], Noiray and Schuermans [5] and Prieur et al. [6] to name a few. However, limited attention is paid for the fixed-point stable regime. Knowledge about stable operation can be important for optimization and online monitoring of annular combustors in their actual operational state. Rouwenhorst et al. [7] published a strategy to identify the azimuthal eigenmode pairs and obtain their decay rates from real-time measurements. The identification strategy was demonstrated on surrogate data only. The aim of this work is to apply the output-only identification on experimental data from an annular combustor, as a validation of the proposed strategy for practical time series.

## METHODOLOGY

Simultaneously measured acoustic signals recorded around the annular combustion chamber circumference are used to identify the dynamics of azimuthal thermodynamic modes. The parameters of a low-order model are fitted to the data, such that the key dynamics is captured well. Modal behaviour, growth rates and frequencies can be determined from the model parameters. The objective is to identify the system in the low-amplitude stable regime, which justifies a linear description of the dynamics.

### Model

Two coupled harmonic oscillators, representing the acoustics, model the thermoacoustic dynamics related to one azimuthal mode order. The two corresponding amplitudes belong to complex harmonic basis functions around the annular geometry with fixed wavelength. Considering the quiescent acoustics of a cylindrically symmetric annulus, they form the eigenspace of the degenerate eigensolution. It is assumed that phenomena that alter the growth rate and frequency, in special due to the thermoacoustic feedback, are very small compared to the acoustic eigenfrequency. This allows linearization around the acoustic solution, decoupling the slow dynamics (amplitude modulations) and the fast dynamics (acoustic oscillations). A system of ordinary differential equations (ODE's) can describe the coupling between the two slowly varying harmonic oscillator amplitudes, including thermoacoustic feedback. Relating the acoustic sensor signals (in analytic form, i.e. including its complex conjugate) to the stochastically forced system, yields the state-space representation in Equation (1).

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{M}\mathbf{x} + \mathbf{w}_{\mathbf{x}} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{w}_{\mathbf{y}} \end{aligned} \qquad (1)$$

The vector $\mathbf{x}$ contains the complex pair of oscillator amplitudes. The complex 2x2 system matrix $\mathbf{M}$ characterizes the system dynamics, forced by the stochastic process $\mathbf{w}_{\mathbf{x}}$ which originates from the turbulent combustion. The output vector $\mathbf{y}$ contains the analytic pressure sensor output signals around the combustor circumference, possibly containing measurement noise $\mathbf{w}_{\mathbf{y}}$. The output matrix $\mathbf{C}$ contains information about the angular sensor positions and the monitored mode order. A nomenclature is attached at the end of the paper.

The system matrix $\mathbf{M}$ describes the evolution of the acoustics, including the feedback loop resulting from the flame response. Amplitudes remain constant for pure acoustics, in which case the system matrix is a null matrix. Damping rates of the two oscillators would be found as real negative values on the diagonal. The anti-diagonal of the matrix establishes coupling of the oscillators. Thermoacoustics and fluid dynamics can contribute to all matrix entries of $\mathbf{M}$, which we will consider as a black box to be identified.

The left column of represents the slow modulation of the two acoustic amplitudes, resulting from the presence of the first acoustic amplitude. Identically, the right column shows the respective modulations because of a finite amplitude of the second acoustic degree of freedom. In the framework of flame transfer functions (FTF's), the four matrix entries are related to the Rayleigh Index based on the FTF's evaluated at the eigenfrequencies of the system. It is noted that these FTF's would describe the heat release response to azimuthal pressure excitations. All matrix and vector entries in Equation (1) are complex, allowing a response to both azimuthal acoustic pressure and velocity fluctuations and resulting in phase relation sensitivity between the two amplitudes.

The eigensolution of this system matrix yields the deviations from the acoustic eigenfrequency and growth rates of the eigenmode pair. The eigenvectors contain the spatio-temporal behaviour of the eigenmodes. Because the system matrix can be a function of its own eigenfrequencies, the eigensolution can be discontinuous for continuous parameter changes. A relatively smooth flame transfer function with moderate gain prevents the latter.

### System Identification

The thermoacoustics as modelled in Equation (1) can be identified in several ways, which was shown in Rouwenhorst et al. [7]. Such in situ identification yields the coupled thermoacoustic dynamics under the prevailing conditions. The pair of eigenvalues of the system matrix $\mathbf{M}$ yield the eigenfrequencies and growth rates (i.e. stability margin) corresponding to the monitored azimuthal mode order.

In this work, two methods are applied to analyse the experimental data. Main motivation to apply two methods is cross-validation, because no expected values or analytical predictions for the thermoacousic system are available. In both methods, a bandwidth of the spectrum is analysed, isolating the spectral peak of interest, which is the first azimuthal mode order in this work. This rectangular filter corresponds to a sinc filter in time domain.

Acoustic signals from two or more sensors around the circumference are required to identify the system. More than two sensors is strongly recommended in order to filter out measurement noise.

From measurements, discrete time series are available, requiring the explicit discrete form of the state-space representation. The eigenvalue solution of the estimated discrete time system matrix is then transformed back to continuous time in order to obtain the eigensolutions of the continuous dynamic system.

#### *Stochastic Subspace Identification*

A Stochastic Subspace Identification (SSI) algorithm has been applied following Tanaka and Katayama [8], in which the entire structure of the state-space representation is identified. An algorithm, including a weighted singular value decomposition of a matrix with time-delayed data sets, estimates the dominant dynamics for the chosen degrees of freedom. As the mathematics goes beyond the scope of this work, we refer the interested reader to the book of Katayama on this subject [9].

It is important to note that the mode shape cannot be prescribed in this method, such that it is not possible to enforce a harmonic basis around circumference and exploit knowledge concerning the relation between sensor positions and azimuthal mode order.

Per sensor, three delayed time series were considered with mutual intervals of 32 samples. Physically, this means that the dynamics is identified on basis of the evolution of all observed states, during a period of about four cycles each.

### *Least Squares Fitting*

In order to be able to prescribe the mode order we want to identify, a more straightforward strategy is applied that makes repeated use of least squares fitting, referred to as LSQ. We choose a basis of clockwise and anticlockwise traveling waves to span the eigenspace of acoustic solutions. For given mode order and at least three measurement locations, the wave amplitudes can now be fitted by taking the Moore-Penrose pseudoinverse of the overdetermined problem.

$$\mathbf{x} = (\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T\mathbf{y} \qquad (2)$$

The analytic representation $\mathbf{y}$ in Equation (2) is obtained by considering the positive frequencies of the measured signals in Fourier domain.

The first line of the state-space representation in Eq. (1) in discrete formulation reads

$$\mathbf{x}(i+\Delta t) = \mathbf{A}\mathbf{x}(i) + \mathbf{v}_{\mathbf{x}}(i) \qquad (3)$$

Again using the pseudoinverse, we estimate the discrete system matrix $\mathbf{A}$. The process noise $\mathbf{v}_{\mathbf{x}}(i)$ is the discrete counterpart of $\boldsymbol{w}_{\mathbf{x}}(t)$. The following matrix $\mathbf{X}$ should be understood as a concatenation of the state vector $\mathbf{x}$ determined in Equation (2), for all available time steps.

$$\mathbf{A} = \mathbf{X}_{i+\Delta t}\mathbf{X}_i^T(\mathbf{X}_i\mathbf{X}_i^T)^{-1} \qquad (4)$$

With sufficient time length of the data, the influence of the unknown process noise can be neglected. However, it must be noted that this two-step identification approach does not yield a bias-free estimate in the presence of measurement noise $\mathbf{w}_{\mathbf{y}}$.

Similarly to SSI, this method identifies the state-space based on a time delay $\Delta t$, which is equally set to the span of about four thermoacoustic cycles. A parameter study on the influence of this time delay parameter has shown that the order of magnitude of this time delay should be picked carefully; after long time delays the correlation between the two states vanishes and too short time delays also result in poor results. The latter is thought to be related to the sinc filter used to isolate the frequency peak and/or the temporal correlation of the combustion noise.

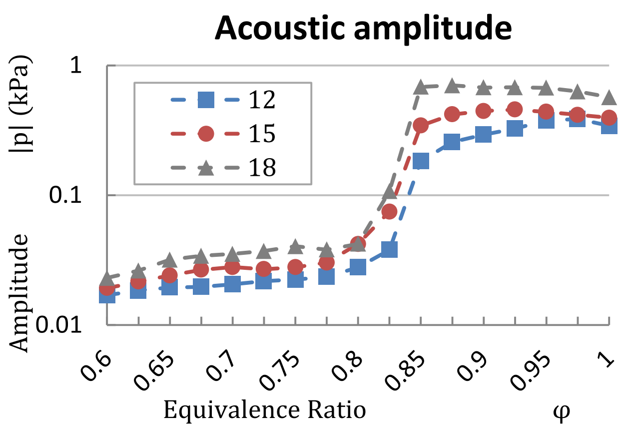


**Figure 1: Root mean square of the acoustic amplitude as a function of equivalence ratio and burner spacing, previously published in [4].**

## EXPERIMENTAL DATA

Data from a laboratory scale annular combustor, kindly provided by J. Dawson and N. Worth, have been used to validate the identification strategy on experimental data. The combustor was designed to investigate azimuthal instabilities, including the effect of burner spacing. Fuel lines transport the perfectly premixed ethylene/air mixture towards the combustion chamber, where swirlers anchor the flames with co-rotating swirl, as often used in industrial applications. The atmospheric rig is prone to instabilities of the first azimuthal mode order, producing high amplitude limit cycle oscillations with a frequency of about 1700 Hz.

For equal flame conditions, the Reynolds number at the swirl burners was kept constant in all cases. As a result, the total heat release increases with the number of burners and increasing equivalence ratio, therewith resulting in an increased speed of sound and natural frequencies.

A more extensive description of the experimental setup and measurement methods, refer to publications of Worth and Dawson [4, 10].

### Measurements

Dynamic pressure was measured in three fuel lines, with 120-degree intervals equally distributed around the combustor circumference. The fuel lines were equipped with two pressure transducers, of which we used the ones closer to the combustion chamber for this work.

Three different burner separation distances were tried, adding up to a respective total of 12, 15 and 18 burners around the combustor annulus, to investigate the effect of flame separation distance. For these three cases, a discrete set of equivalence ratios was tested ranging between $\phi = 0.6$ and $\phi = 1$. Measurements were performed under steady operation with a sampling rate of 30 kHz for a period of 4.36 seconds.

The combined root mean square of the three pressure transducers for the three burner separation cases is shown in Figure 1. For every burner spacing, the acoustic amplitude is low for equivalence ratios $\phi \leq 0.8$ and high for equivalence ratios $\phi \geq 0.85$, marking the transition from stable operation to limit cycle operation. The amplitude of the acoustics roughly increases an order of magnitude, corresponding to a factor 100 for the acoustic energy.

With the increasing equivalence ratio, the flame response will change. In first instance, the phase and gain are likely to be altered by the different fuel mixture. In addition, the temperature increase in the combustion chamber should be addressed, which can drastically change the burner impedance, depending on the acoustic characteristics of the plenum and fuel lines.

## IDENTIFICATION RESULTS

In this work, the aim is to identify the stability of the system as a whole, rather than discriminating between the phenomena that lead to the instability. The two identification methods briefly described in the section *Methodology* are applied to the data to obtain the global characteristics of the system, such as the damping rates and dynamic structure, which are contained in the eigensolution.

The bandwidth used for identification, separating the first azimuthal mode from other spectral content, is fixed between 1200 and 2200 Hz in all cases. A waterfall plot with the power spectral density in this bandwidth is shown in figure 2 for the case of 12 burners. Especially for low equivalence ratio, a considerable portion of the power is not related to the peak of interest. It must be noted that the choice to keep the same bandwidth and also the time delays chosen in the identification methods can have a considerable influence on the quantitative results.

### Damping ratio

The growth rate that is recovered by the real part of the eigenvalue of the system matrix, is rewritten to a damping ratio of a harmonic oscillator. To this end, the negated growth rate is divided by the eigenfrequency (in radians), which is given by the imaginary part of the eigenvalue.

$$\zeta_i = -\Re(\lambda_i) / \Im(\lambda_i) \qquad (5)$$

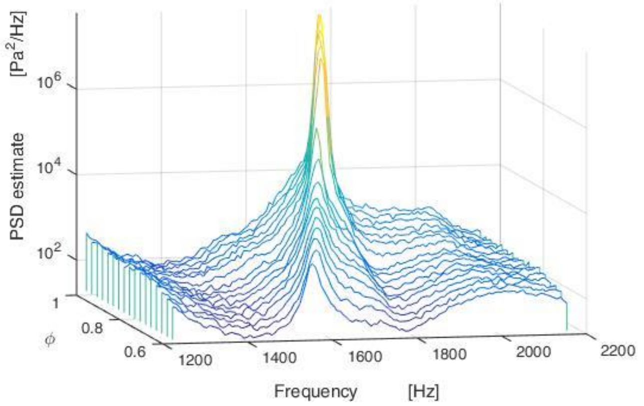


**Figure 2: Power spectral density (PSD) as a function of equivalence ratio in the analysed bandwidth for the case with 12 burners.**

Figure 3, 4 and 5 show the identified damping ratios for the case with 12, 15 and 18 installed burners respectively. The results show that the system has a pair of damping ratios at low equivalence ratios, that vanishes between $\phi = 0.8$ and $\phi = 0.85$. The damping ratio does not become negative because the system quickly develops to a limit cycle, rather than showing continuous exponential growth.

The pair of damping ratios is for many operating conditions clearly distinct, indicating split eigenvalues and a dynamic system that is not (exactly) degenerate. Especially for the case with 18 burners, the stability margins show an interesting behaviour with strong splitting of the growth rate.

Even though the effect of eigenvalue splitting is very small in comparison to the eigenfrequency, it cannot be neglected with respect to the stability margin. This is directly related to the damping ratio, which is typically strongly underdamped for azimuthal thermoacoustic eigenmodes.

The pairs of fixed point stable eigenmodes always co-exist, being excited by the combustion noise. This stands in contrast to limit cycle oscillations, in which an attractor has a unique location in the phase space, see for example Noiray et al. [11]. Therefore, only one solution is observed (possibly depending on initial conditions), unless the stochastic forcing is quite large with respect to the limit cycle amplitude.

For low equivalence ratio, the two methods return quite different damping ratios. It is observed that the damping values for SSI approach the values for of LSQ when the peaks are isolated more carefully from other spectral content. SSI is more susceptible to dynamics that is unrelated to the monitored mode, because the mode order is not prescribed and knowledge of the sensor positions is not exploited. This shows that LSQ is more robust, because less parameters are fitted. A window with optimized bandwidth moving with the monitored frequency could significantly improve the identification process.

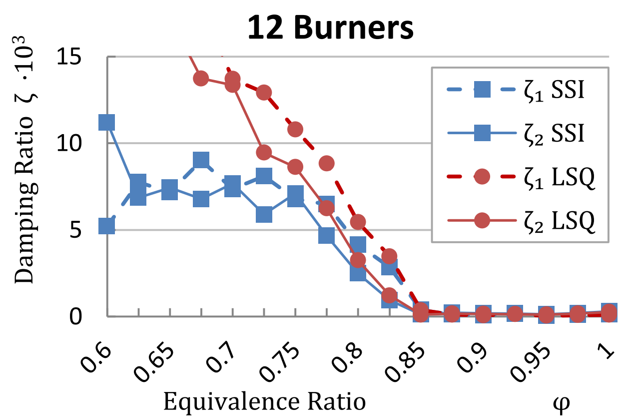


**Figure 3: Pair of identified damping ratios as a function of equivalence ratio, for the first azimuthal mode order for the case with 12 burners.**

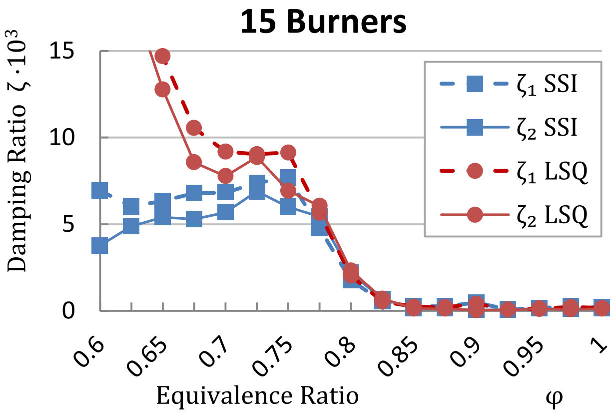


**Figure 4: Pair of identified damping ratios as a function of equivalence ratio, for the first azimuthal mode order for the case with 15 burners.**

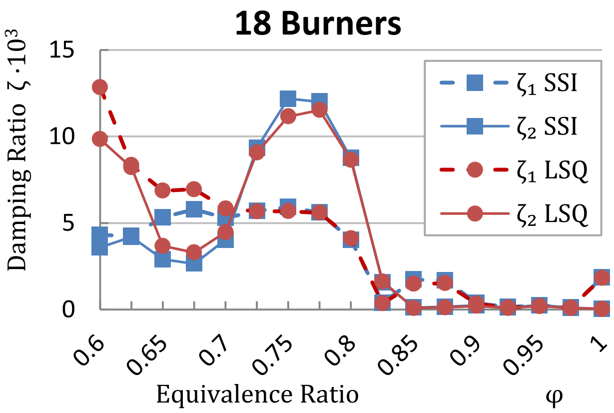


**Figure 5: Pair of identified damping ratios as a function of equivalence ratio, for the first azimuthal mode order for the case with 18 burners.**

## Eigenvectors

At every operation point, two eigenvalues are obtained by solving the eigenvalue problem. The eigenvalue pairs of two adjacent operation points were connected by maximizing the inner product of their corresponding eigenvectors. This way the connected lines in the plots in Figures 3 to 7 point to the most similar mode shape, yielding relatively continuous descriptions for the damping ratios and eigenfrequencies as a function of the equivalence ratio.

In Table 1, some eigenvector pairs for the case of 12 and 18 burners are presented in the form of two amplitudes $|F|$ and $|G|$ and their mutual phase difference $Ang$. The amplitudes $|F|$ and $|G|$ correspond to the eigenvector contributions of the two waves traveling around the circumference in opposite direction. When they have a similar magnitude, the mode is a standing wave. In that case, the phase difference between the waves ($Ang$) is of interest, as it determines the angular standing wave orientation. A set of predominantly standing wave solutions is found for the case with 12 burners, which can be seen from the similar amplitudes of $|F|_2$ and $|G|_2$. In case of 18 burners, predominantly traveling waves are observed; refer to the negligible contribution of $|G|_2$. Data of more stable operation points are not shown, because the eigenvectors are not very coherent from point to point. This lack of two clear distinct eigensolutions can be attributed to the proximity to a degenerate system and the relatively short data sets.

## Eigenfrequencies

With the increasing equivalence ratio, the eigenfrequency rises. This can be expected to play an important role in the stability of the thermoacoustic system. In Figure 6, the identified eigenfrequencies are shown in the case of 18 burners. The splitting of the eigenvalues we have seen in the damping ratio is, to a lesser extent, also observed in the eigenfrequencies.

## Flame-to-flame distance

The amount of burners has a pronounced effect on the stability, in a non-trivial manner. In the case with the closest flame spacing (18 burners), the damping of one of the eigensolutions varies non-monotonically with as a function of the equivalence ratio. On basis of the results, the behaviour cannot be explained in detail, but we will hypothesize a reason for the qualitative behaviour observed.

In the case of 12 burners, the flames barely interact directly. The swirlers cause individual vortices with limited influence of the global flow field. Due to unavoidable minor imperfections in the experimental setup, the cylindrical symmetry is broken, splitting the eigenmodes in two (predominantly) standing wave solutions.

For close burner spacing (18 burners), the flames are merging. This happens in a directional manner, because the swirlers are co-rotating. To get an insight in the complex heat release pattern in this case, please refer to the publication of Worth and Dawson [10] on this topic. One can imagine this can cause an increased flame response to azimuthal particle velocity. Additionally, a global mean flow field develops around the circumference because the vortices on top of the swirlers start to merge, which can lead to eigenfrequency splitting of the acoustic subsystem. These effects can promote traveling wave solutions for the (fixed point stable) azimuthal thermodynamics.

The case with 15 burners seems to behave in an intermediate state between the system with individual burners and the system with flame-flame interaction.

| Case | $\phi$ | $\|F\|_1$ | $\|G\|_1$ | $Ang_1$ | $\|F\|_2$ | $\|G\|_2$ | $Ang_2$ |
|---|---|---|---|---|---|---|---|
| 12 | 0.75 | 0.23 | 0.97 | 0.25 | 0.87 | 0.50 | 1.08 |
| | 0.775 | 0.34 | 0.94 | -1.01 | 0.68 | 0.73 | 0.70 |
| | 0.8 | 0.65 | 0.76 | -0.47 | 0.74 | 0.67 | 1.64 |
| | 0.825 | 0.34 | 0.94 | -0.82 | 0.74 | 0.67 | 1.85 |
| 18 | 0.75 | 0.22 | 0.98 | -0.40 | 1.00 | 0.09 | -0.44 |
| | 0.775 | 0.38 | 0.92 | -1.63 | 1.00 | 0.07 | -2.81 |
| | 0.8 | 0.24 | 0.97 | -1.80 | 1.00 | 0.04 | 0.14 |
| | 0.825 | 0.34 | 0.94 | -1.63 | 0.98 | 0.18 | 1.77 |

**Table 1: Identified eigenvectors for equivalence ratios shortly before instability, for the distant (12) and close (18) burner separation using LSQ. One eigenvector is represented by the amplitudes |F| and |G| and their phase difference Ang.**

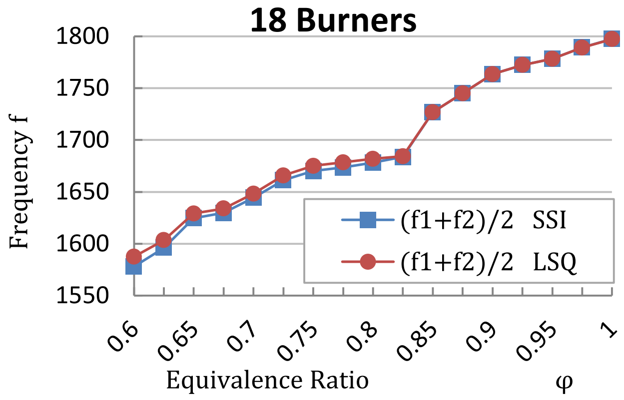


**Figure 6: Mean eigenfrequency of the mode pair as a function of equivalence ratio for 18 burners.**

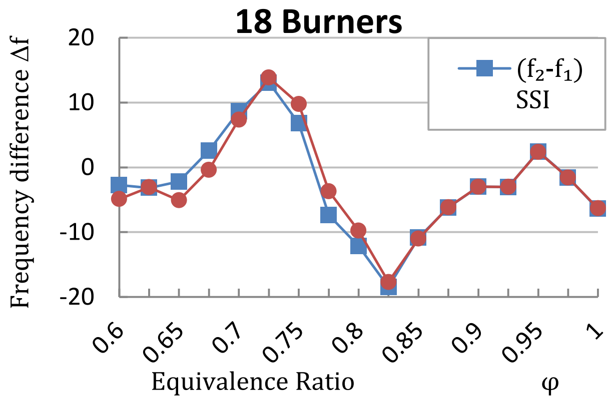


**Figure 7: Difference in eigenfrequency of the mode pair as a function of equivalence ratio for 18 burners.**

## MONITORING STRATEGY

The results show that the proposed identification method can be applied to annular combustors that are prone to azimuthal instabilities. In practice, this can serve as a stability margin that can enhance flexible operation of industrial machines by constant monitoring of potentially hazardous azimuthal eigenmodes.

Such in situ stability determination is beneficial compared to a stability map, in that it reflects the current state of the machine and operation conditions. Effects due to azimuthal velocity and flame-flame interaction are hard to predict, without performing measurements on a full annular setup. Such details are not easily included in a low-order stability analysis, but its effect on the stability can directly be identified, using the in situ state-space representation of the dynamics.

It must be noted that the stability margin (damping ratio) is not necessarily continuous, for continuously changing system parameters, in case that the flame transfer functions change strongly as a function of frequency. In that case, the system matrix depends on its own eigensolution, i.e the eigensolution is of transcendent nature. Furthermore triggering can occur, due to nonlinear response of the heat release to the finite acoustic amplitudes. For these reasons, instability might occur before the identified damping reaches zero, which addresses the need for a certain safety margin in practical applications.

## CONCLUSIONS

In previous work, it was shown that azimuthal thermoacoustic dynamics found in annular combustors are effectively described using a complex two-dimensional state-space representation (representing a four-dimensional system). In addition, it was shown that a quantitatively accurate identification of the dynamics can be carried out, for surrogate data which is generated on basis of this model description. Assertion that the identification strategy also works successfully on experimental data had not yet been delivered, which is now provided in this work.

The output-only identification strategy has been performed on experimental data of a laboratory scale annular combustor. Qualitatively, the results are promising; consistent damping ratios and modal frequencies are obtained for the rather short experimental data sets and the pair of damping ratios decrease towards zero before instability occurs and high amplitude limit cycle oscillations are found. The eigenvalue pairs are mostly very similar, yet some structured deviations from a degenerate system are found, which was expected for an experimental setup.

Two different identification methods have been used to obtain the system characteristics of the thermoacoustic dynamics. The results of the two methods are very similar for higher equivalence ratios, increasing the confidence in the results. For low equivalence ratio, LSQ identifies a higher stability margin compared to SSI. It seems like SSI responds to dynamics that is unrelated to the first azimuthal mode order, which shows the robustness of LSQ by the use a prescribed output matrix.

The case with close burner spacing (18 burners) shows predominantly traveling wave solutions, with strong splitting of the eigenvalues. Although the physics cannot be explained on basis of the results, the effect it has on both the stability and eigenfrequency splitting is captured in detail. We hypothesize that the flame response to azimuthal acoustic excitation and the mean flow field in the annulus contribute to the eigenvalue splitting.

It is observed that eigenvalue splitting can become relevant for vanishing system stability, even though the magnitude is negligible with respect to the eigenfrequency. This observation stresses the necessity to decompose the signals on a two-dimensional basis, allowing for the typical azimuthal solutions as a mix of traveling and standing waves. In less controlled environments of industrial applications, eigenvalue splitting is expected to be more pronounced.

In the first place, the identification can serve as a stability margin measure to monitor annular combustion systems during operation. When the isolation of the spectral peak from other dynamic content is optimized, results can be compared in a quantitative way. In testing, the method enables quantitative comparison of the thermoacoustic behaviour, when changes or upgrades are made to the machine.

## NOMENCLATURE

| | |
|---|---|
| $\zeta$ | Damping ratio |
| $\boldsymbol{\lambda}$ | Eigenvalue |
| $\phi$ | Equivalence ratio |
| $\Im(.)$ | Imaginary part |
| $\Re(.)$ | Real part |
| **A** | Discrete system matrix |
| Ang | Angle between waves F and G |
| **C** | Output matrix |
| f | Frequency |
| F | CW acoustic wave |
| G | ACW acoustic wave |
| **M** | System matrix |
| **v** | Discrete noise vector |
| **w** | Noise vector |
| **x** | State vector |
| **X** | Matrix of concatenated state vectors |
| **y** | Output vector |

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