

## **An Improved Threshold Determination Method Based on PSO Algorithm for Sensor Fault Diagnosis**

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### **ABSTRACT**

Based on the Kalman filter theory, detecting sensor faults in gas turbine control system and performing output reconstruction for fault signals is a hot focus in recent years. The selection of detection thresholds directly affects the rate and accuracy of fault diagnosis, which is extremely important for the safe operating of control system and its value is usually determined by actual experience and the noise characteristics of the sensor itself. Based on characteristics of the sensor and the observed signals, the cost function of the threshold is constructed by the combination of the leakage alarm probability and the false alarm probability, which is then used as the fitness function in the particle swarm optimization (PSO) algorithm, and the inertia weight in the iterative search process is adaptively changed. So, an improved dynamic weight adaptive threshold determination method based on PSO algorithm comes into being. The simulation comparison of the proposed method with the traditional threshold determination method shows that the timeliness, accuracy and integrity of the method is improved somewhat.

### **1. INTRODUCTION**

The control system of gas turbine works on the basis of sensor measurement signals. However, the sensor usually works in a harsh environment such as high temperature and high pressure, making the sensor malfunctioning component(Garg, 2004). In order to improve the reliability

and safety of the control system, it is very important to diagnose and isolate the sensor faults.

There are various methods for sensor fault diagnosis. The most widely used and most classic method is the threshold method(Beattie et al., 1983), that is, a fault indication function is formed according to the residual, which is compared with the fault detection threshold. If the fault indication function is smaller than the predetermined threshold, the sensor is considered to be fault free. Otherwise, if it is greater than the predetermined threshold, the sensor is identified faulty. It can be seen that the reasonable fault detection threshold is the key factor for the accuracy of sensor fault diagnosis(Emami-Naeini et al., 1988). At present, the fault detection threshold is usually determined according to the rule that the fault indication function obeying the chi-square distribution, In addition, some scholars have proposed new threshold determination methods and theories. B.K. Walker(Walker and Gait, 2015) and E. Gai(Gai et al., 2007) established a dynamics model for the fault diagnosis process and derived a time-varying threshold based on the theory of Markov process. X. Ding, P. M. Frank, L. Guo and et al(Ding et al., 1993, Ding and Guo, 1998) proposed a method for calculating the minimum detectable fault in the frequency domain, and the threshold was set only slightly higher than the residual evaluation function that corresponded to the minimum detectable fault. And adaptive thresholds were proposed by R. J. PATTON(Patton et al., 1989), which means that thresholds

are varied according to the control activity, noise and the fault signal properties of the monitored system. But it must take into account the dynamic effects of time-varying measurement statistics and the cumulative effects of error probabilities in dynamic process. Therefore, the selection of practical adaptive laws is also a difficulty point.

There are two kinds of errors if the threshold selected improperly, one is that the fault has occurred, but the system is still considered to be normal, and there is no alarm signal, which is called missed alarm error. The other is that the system works normally, but it is considered to be faulty, and a warning signal is sent out, which is called a false alarm error. Therefore, the cost function of fault threshold is the tradeoff of the false alarm and the missed detection rate, with which the threshold is constructed to minimize the false alarm rate and the missed detection rate during the fault diagnosis process.

Particle swarm optimization (PSO) algorithm is an emerging intelligent optimization technology, with local optimization and global optimization search ability. In this paper, the PSO algorithm is proposed to find the optimal threshold for the cost function. Select several training samples of sensor measurement noise as the initial population of PSO, and then calculates the optimal fault detection threshold iteratively. Compared with the traditional threshold selection method, it is found that the fault diagnosis performance of this method is better.

## 2. THE THRESHOLD DETERMINATION ALGORITHM DESCRIPTION

At present, the method of probability statistics is the main method to determine the diagnosis threshold. Although it is reasonable to determine the threshold by the knowledge of probability theory, it is greatly stochastic. Therefore, this section will describe the improved threshold determination method based on PSO algorithm: the key steps and the calculation process.

### 2.1 THE BSAIC THEORY OF THE IMPROVED ALGORITHM

The improved method is divided into two parts. Firstly, based on statistical knowledge, the cost function is constructed. Then, the cost function is used as the fitness function of the PSO algorithm to optimize iteratively.

#### 2.1.1 THE THRESHOLD DETERMINATION METHOD BASED ON COST FUNCTION(FRANK, 1990)

Assuming that the actual measurement value of the sensor is  $y(t)$  and the estimated signal of the Kalman filter is  $\hat{y}(t)$ , which obeys the normal distribution with mean is  $\mu$ , and variance is  $\sigma^2$ , then the probability distribution density function satisfies:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (1)$$

Where the mean  $\mu$  represents the amplitude of the true signal, and the variance  $\sigma^2$  is decided by various noises.

Assuming that the residual  $z = y(t) - \hat{y}(t)$ , which obeys the normal distribution with mean is 0, and variance is  $2\sigma^2$ . Then the probability density function of  $z$  obtained:

$$f(z) = \begin{cases} \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{z^2}{4\sigma^2}} & z > 0 \\ 0 & z \leq 0 \end{cases}. \quad (2)$$

If the fault detection threshold is  $h$ , the false alarm rate can be expressed as:

$$P_{e2} = \int_h^{\infty} f(z) dz = \int_h^{\infty} \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{z^2}{4\sigma^2}} dz. \quad (3)$$

Then convert the above formula to  $u_1 \sim N(0,1)$ , which means:

$$P_{e2} = \frac{1}{\sqrt{2\pi}} \int_{\frac{h}{\sqrt{2}\sigma}}^{\infty} e^{-\frac{u_1^2}{2}} du_1. \quad (4)$$

$$\text{Where } u_1 = \frac{z}{\sqrt{2}\sigma}.$$

When the sensor fails, the mean value  $y(t)$  changes from original  $\mu$  to  $\mu + B$ , and  $B$  is the fault deviation, and the noise variance remains unchanged. The probability distribution density function of the corresponding residual  $z$  is:

$$f(z/B) = \begin{cases} \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{(z-B)^2}{4\sigma^2}} & z > 0 \\ 0 & z \leq 0 \end{cases}. \quad (5)$$

Therefore, the missed detection rate can be expressed as:

$$P_{e1} = \int_0^h f(z/B) dz = \int_0^h \frac{1}{2\sqrt{\pi}\sigma} e^{-\frac{(z-B)^2}{4\sigma^2}} dz. \quad (6)$$

Again, converted the above formula to  $u_2 \sim N(0,1)$ , which is:

$$P_{e1} = \frac{1}{\sqrt{2\pi}} \int_0^{\frac{h-B}{\sqrt{2}\sigma}} e^{-\frac{u_2^2}{2}} du_2. \quad (7)$$

$$\text{Where } u_2 = \frac{z}{\sqrt{2}\sigma}.$$

In order to determine the ideal fault detection threshold  $h$ , suppose the sensor's failure probability be  $P_f$ , introducing a cost function:

$$C(h) = P_f P_{e1}(h) + (1 - P_f) P_{e2}(h). \quad (8)$$

This cost function means that the ideal threshold  $h$  should make the false alarm rate and the missed detection rate of the fault diagnosis as small as possible, so the ideal fault detection threshold can be obtained by finding the minimum value of the cost function.

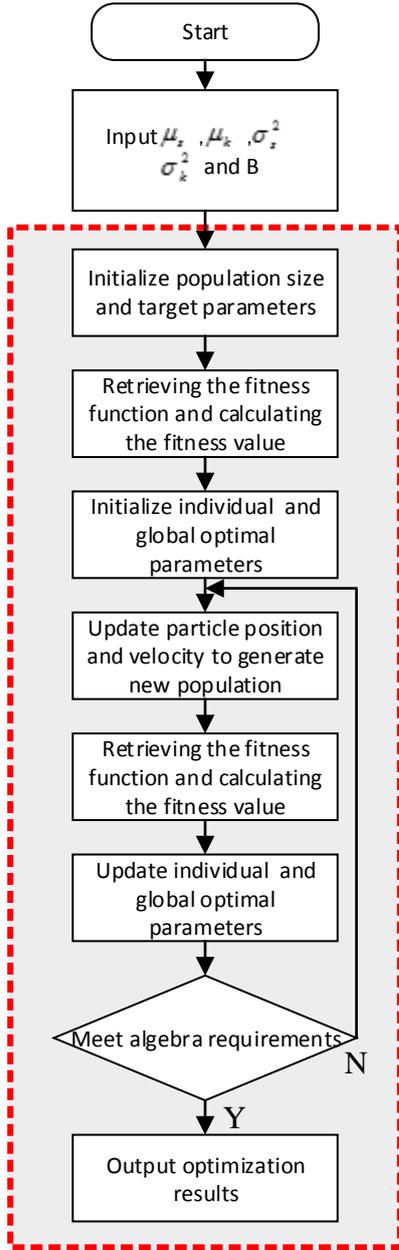
#### 2.1.2 PARTICAL SWARM OPTIMIZATION ALGORITHM

PSO(Kennedy, 2011) algorithm is initialized into a group of random particles (random solutions), and then iteratively finds the optimal solution. In each iteration, the particle updates itself by tracking two extreme values, the first is the optimal solution found by the particle itself, which is called the individual extreme value. The other extreme

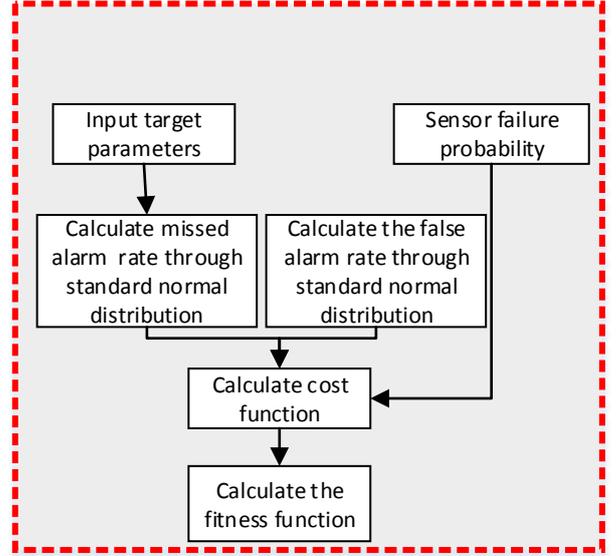
value is the optimal solution currently found for the entire population, which is called the global extreme value.

## 2.2 OBTAIN THE OPTIMAL THRESHOLD BY PSO ALGORITHM

As is shown in the figure 1, for each sensor, the optimization program calls the cost function as the fitness function by inputting mean value and variance of sensor measurements and Kalman filter observations, and fault deviation. In the PSO algorithm, set the population size  $S = 50$ , the maximum number of iterations  $G = 100$ , the learning factor  $c_1 = 1.3$ ,  $c_2 = 1.7$  and the linear decreasing inertia weight are used. The update target of the optimization process is that the fitness function reaches its minimum.



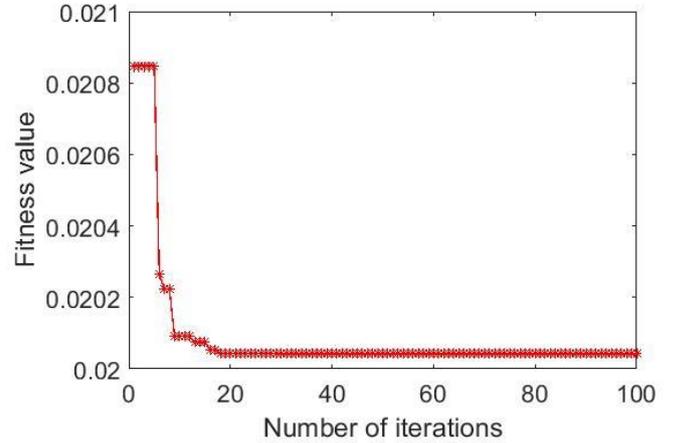
(a)PSO target parameters



(b)Fitness function

**Figure 1. Flow chat of obtaining the optimal threshold by PSO algorithm**

Taking the rotational speed sensor as an example, the fault detection threshold of the rotational speed sensor is optimized at the steady point, where  $P_n = 100\%$ . The cost function of the sensor is reserved as the fitness function, and the PSO algorithm is utilized to solve the problem. The fitness function iteration curve is shown in Figure 2. It can be seen that the fitness function basically tends to be stable after the number of iterations exceeds 40 times, indicating that the algorithm has searched the optimal solution, and the optimal threshold  $h_n$  of the rotational speed sensor gained.



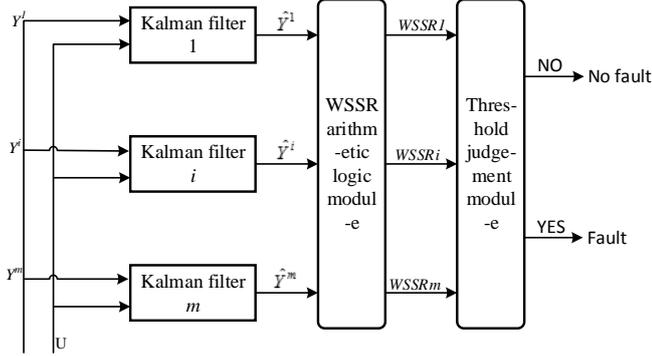
**Figure 2. Iteration curve of fitness function**

Similarly, the PSO algorithm can be utilized to determine the optimal detection threshold corresponding to other sensors. After solving the optimal fault detection threshold  $h_{opti} = (h_n, h_{P_2}, h_{P_3}, h_{T_3}, h_{T_4})$  of all the sensors, the weighted square sum of the threshold is performed according to the processing procedure of the sensor fault indication signal. And then the best sensor failure detection threshold appears:

$$\varepsilon_{opti} = (h_{opti})^T [\Sigma]^{-1} h_{opti}. \quad (9)$$

### 3. NUMERICAL EXAMPLE

In order to verify the feasibility and effectiveness of the fault detection threshold determination method mentioned above, a fault diagnosis simulation experiment was carried out on the MATLAB platform for single sensor fault and multiple sensor faults. Since multi-fault diagnosis is based on single fault diagnosis, limited to space limitation, this paper will take single fault diagnosis as an example to study the optimal fault threshold performance under three conditions: no fault, soft fault and hard fault. The fault diagnosis structure of a single sensor is shown in Figure 3.



**Figure 3. Single sensor fault diagnosis based on Kalman filter bank**

Each filter produces a set of residuals:

$$e^i = Y^i - \hat{Y}^i \quad (i = 1, 2, \dots, m) \quad (10)$$

Where  $Y^i$  and  $\hat{Y}^i$  are the actual measured values of the sensor and the estimated output value of the Kalman filter. The weighted squared sum of the residual sequences is calculated by:

$$WSSR_i = (e^i)^T [\Sigma^i]^{-1} e^i \quad (i = 1, 2, \dots, m) \quad (11)$$

Where  $\Sigma^i = \text{diag}(\sigma^i)^2$ ,  $\sigma^i$  is the standard deviation of the measurement signals for each sensor. And the criteria for determining the occurrence of a single sensor failure are:

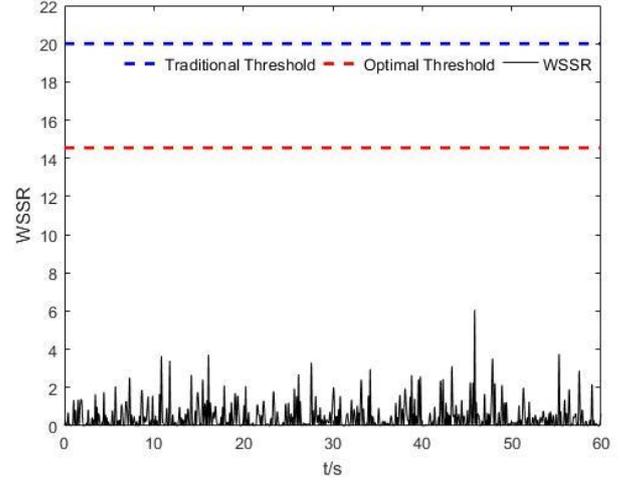
$$WSSR_i = \min(WSSR) \quad \& \quad WSSR_j > \varepsilon \quad (12)$$

$$j = 1, 2, \dots, m (j \neq i)$$

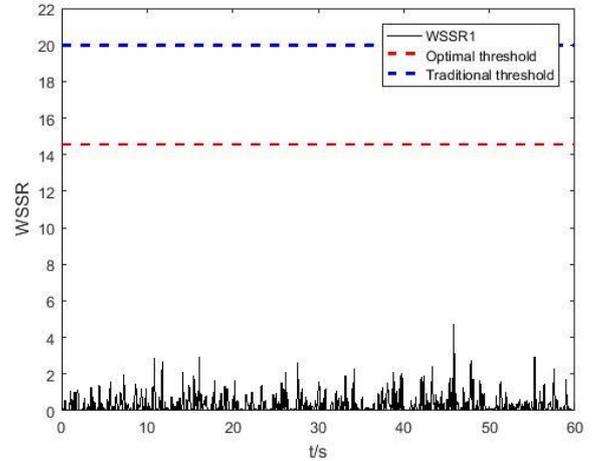
Where  $\varepsilon$  stands for threshold.

In the traditional threshold determination method, threshold sizes are initially determined from the standard deviation magnitudes of the noise and are then increased to account for modeling errors. In the previous study (Merrill et al., 1988), thresholds are determined empirically, which indicates that the hard fault detection threshold values are twice the magnitude of these adjusted standard deviations. With this method, we make a comparison of failure detection times with our optimal threshold for hard and soft faults. Figure 4, 5 and 6 shows the difference for each fault with simulation.

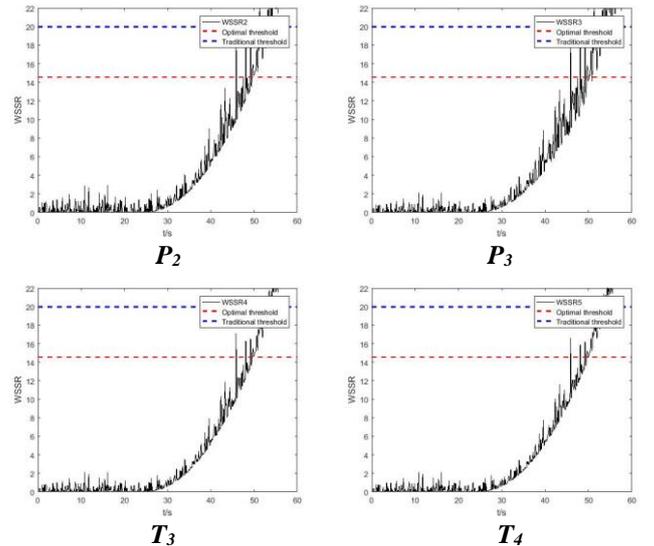
Figure 4 shows the threshold comparison of sensor when there is no fault. The red line is the optimal threshold and the blue line is the traditional method threshold (NASA method). The former is obtained by optimizing the cost function, while the latter is mainly based on the statistical characteristics of noise and then combined with the experience level.



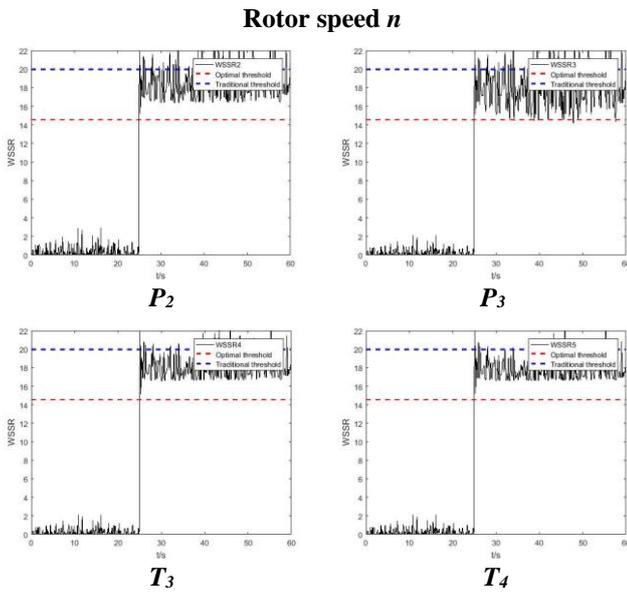
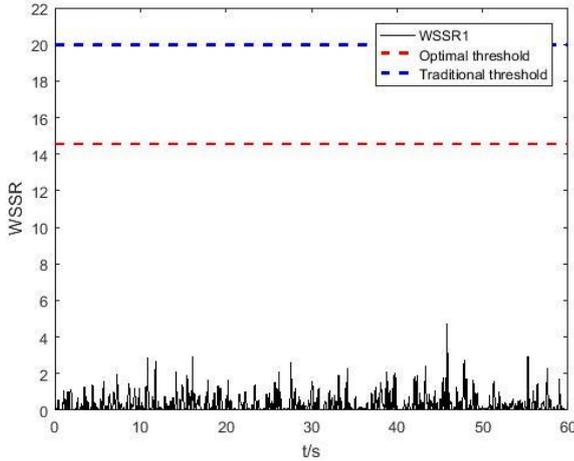
**Figure 4. Threshold comparison with no fault**



**Rotor speed  $n$**



**Figure 5. Threshold comparison with soft fault diagnosis of speed sensor at a steady point**



**Figure 6. Threshold comparison with hard fault diagnosis of speed sensor at a steady point**

When the gas turbine works steadily at design point  $P_n = 100\%$ , set a 2% fault drift rate for the speed sensor at  $t=25s$ , whose fault diagnosis situation is shown in figure 5, and a 0.8% offset fault, as shown in figure 6.

It can be seen from the figure 5 and figure 6, that during the steady state operation of the gas turbine, only the WSSR value corresponding to the speed sensor is always less than the set fault detection threshold. While the WSSR values of other sensors are larger than the fault detection threshold at the time of and after the fault occurs. According to the single sensor fault diagnosis logic, it indicates that only the sensor corresponding to the WSSR1, that is, the speed sensor has failed.

From the figure 5, we know that the speed sensor starts to drift at  $t=25s$ , and the diagnostic system (optimal threshold) detects the sensor failure about 50s. The diagnostic effect has a certain hysteresis, but it does not affect the final diagnosis result. And both kinds of thresholds can identify the fault, compared with the traditional method, our optimal threshold can identify the fault earlier and more accurately, which provides a better basis for the

implementation of fault isolation and fault-tolerant control algorithms.

From the figure 6, we know that the diagnostic system detects the fault as soon as it occurs. In general, both methods can identify the all faults, but the difference is still quite obvious. The optimal threshold can identify the fault all the time, but the traditional method may not in some cases.

Although the optimal threshold has the same effect as the traditional method in some aspects, it still has irreplaceable advantages in terms of accuracy and rapidity. It is worth noting that the selection of the optimal threshold takes into account the model error, noise level and the sensor characteristics. In fact, when the situation has changed, the diagnostic method can also isolate and diagnose the fault effectively.

#### 4. CONCLUSIONS

In order to diagnose the sensor fault efficiently and systematically, this paper proposes a PSO-based threshold determination method. By using the mathematical statistics knowledge, the method combines the mean value, variance, and fault deviation of the sensor and Kalman filter to participate in optimization iteration. With the PSO optimization algorithm, we can get a more reliable threshold. The simulation results also show that, this method has better superiority in false alarm rate and diagnosis speed, compared with the traditional threshold determination method, which introduces the optimization algorithms for fault diagnosis threshold determination.

#### NOMENCLATURE

$\mu_s$	Mean value of sensor measurements
$\mu_k$	Mean value of Kalman filter observations
$\sigma_s^2$	Variance of sensor measurements
$\sigma_k^2$	Variance of Kalman filter observations
$n$	Rotor speed
$P_2$	Pressure of compressor outlet
$P_3$	Pressure of combustor outlet
$T_3$	Temperature of combustor outlet
$T_4$	Temperature of turbine outlet
$P_n$	Normalized speed
$h_n$	Optimal threshold of the rotational speed
$h_{p_2}$	Optimal threshold of compressor outlet pressure
$h_{p_3}$	Optimal threshold of combustor outlet pressure
$h_{T_3}$	Optimal threshold of combustor outlet temperature
$h_{T_4}$	Optimal threshold of turbine outlet temperature
WSSR	Weighted sum of squared residuals

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