

## INFLUENCE OF STEAM INJECTION ON THE STABILITY OF A CENTRIFUGAL COMPRESSOR WITH VANED DIFFUSER

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### ABSTRACT

This paper presents a twofold study on a centrifugal compressor with vaned diffuser and downstream collector. One is on the evaluation of experimental results for both steady and unsteady flow conditions. The other is about the assessment of surge limit extension by means of steam injection at the diffuser inlet. Specifically, the stage stability analysis and effects of steam injection on both performance and stability limit are evaluated and reported based on experimental results.

As the first step, a meanline analysis based on empirical correlations is utilized to evaluate which compressor component dominates the stability (surge) limit. Then surge extension methods are put forward on the investigated compressor to enhance field operability.

To validate the meanline analysis, the static performance of both stage and components from the experiments was analyzed and the root cause of system surge was again given based on the classic stability theory [1].

The experimental results not only prove the validity of the former stability analysis but also extend the surge margin. During the experiments, it was also found that the surge margin improvement was not in proportion to the flow injected. In certain cases, the injected steam at large flow rate can even trigger system surge earlier.

To the authors' knowledge, this is the first report on surge extension with steam injection in open literature.

*Keywords:* steam injection, centrifugal compressor, vaned diffuser, stability

### NOMENCLATURE

$p$  = pressure (kPa)  
 $b$  = axial width (mm)  
 $r$  = radius (mm)  
 $f_s$  = sampling Frequency (kHz)  
 $Q$  = volumetric flow rate (m<sup>3</sup>/s)

$T$  = temperature (K)  
 $C$  = absolute velocity (m/s)  
 $W$  = relative velocity (m/s)  
 $U$  = impeller circumferential velocity (m/s)  
 $n$  = impeller rotational speed (rpm)  
 $N$  = number of blades (-)

### Greek Symbols

$\alpha$  = absolute flow angle from tangential direction (°)  
 $\beta$  = blade angle from tangential direction (°)  
 $\phi$  = flow coefficient defined as  $Q/(\pi r_2^2 u_2)$   
 $\psi$  = pressure rise coefficient defined as  $\Delta P / \frac{1}{2} \rho_\infty u_2^2$

### Subscripts

0 = stage inlet  
1 = impeller inlet  
2 = impeller outlet/vaned diffuser inlet  
3 = vaned diffuser outlet  
4 = collector outlet  
 $r$  = radius direction  
 $\theta$  = tangential direction  
 $\infty$  = atmospheric condition

### INTRODUCTION

Compressor flow instabilities may occur if the fluid flow decreases below a certain limit. Rotating stall and surge are such aerodynamic instabilities that occur in both axial and centrifugal compressors [1-21]. Surge, however, can result in severe vibration and damage to both compressor units as well as reduced efficiency. Even many researches published in open literature on these two phenomena, the prediction and control of surge is still far away from the requirement of industrial application. In most cases, this can only be done case by case as it relies on experiences.

For impellers with vaned diffuser, most of the researches attribute rotating stall and surge to the increase of incidence angle at low flow rate but Spakovszky [4] pointed out that vaneless space just in front of vaned diffuser is the root

cause of rotating stall in NASA CC3. Schleer et al [3] measured the trajectories of tip clearance vortex at different flow rate and concluded that tip clearance vortex caused the surge incipience. More recently, researchers from VKI (von Karman Institute) and University of Hannover [2, 12] noticed the influences of volute on both performance and instability.

To control rotating stall and surge in centrifugal compressor, various design features have been: self-circulation device [14], variable inlet guide vanes (VIGV), adjustable vaned diffuser vanes, ported shroud [16], grooved diffuser [17], air injection or bleeding [8].

To sum up, researches in centrifugal compressor flow instabilities are still not enough for the fully understanding and controlling of rotating and surge.

Recently, with the stringent requirements on pollutant emissions, the Chinese government encourages distributed energy system especially combined cooling, heating and power system (CCHP) in which the key component is a gas turbine. Considering the population of residential areas in most of Chinese cities, it has been found that a gas turbine of 2MW output level suites best for CCHP system. At this power level, the gas turbine efficiency is much lower than those in power plants. Generally, small gas turbines are always used together with a HRSG (Heat Recovery Steam Generator) which could provide heating in winter and cooling in summer with absorption refrigerating machine.

In most part of China, however, there are almost no requirements on both cooling and heating in transitional seasons which mean that gas turbine has to be shut down due to its low simple cycle efficiency.

Under such background, the authors intend to boost the power output and efficiency through re-inject the steam from HRSG. However, to fulfill this aim, the compressor surge line has to be moved to the left due to the increased back pressure caused by steam injected. According to the prior theoretical simulation, the surge margin should be improved at least 8%. Then a question arises: can we use part of the power boost steam to increase the surge margin without using an external air supply?

To answer the question, and as first step, the available compressor range and root cause of surge needed to be identified which finally determines the control strategy. The original compressor includes a radial impeller ( $b_2=90^\circ$ ) and vaned diffuser.

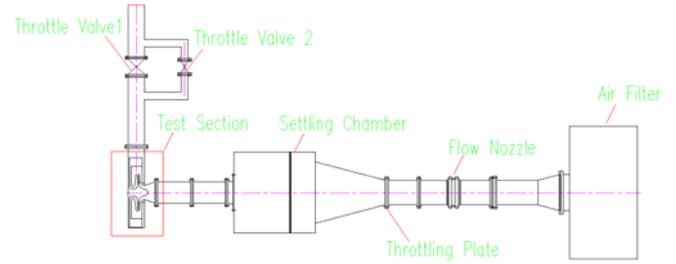
In order to clearly identify the influence of isolated impeller and vaned diffuser, the same impeller was tested with both vaneless and vaned diffuser. Results of the first combination have been reported in [17].

The current paper reports related experimental results about impeller and vaned diffuser in phase 2 of the project. Structure of the paper could be summarized as below, a brief introduction was provided in section 1; test facility, experimental procedure, and data acquisition were illustrated in section 2; stability analysis, stage and subcomponent performance, and some surge characteristics were analyzed in section 3; influences of steam injection characteristics at different rotational speed on stage performance and surge range in section 4; conclusion and acknowledgement were given in the last two sections.

## TEST FACILITIES AND INSTRUMENTATION

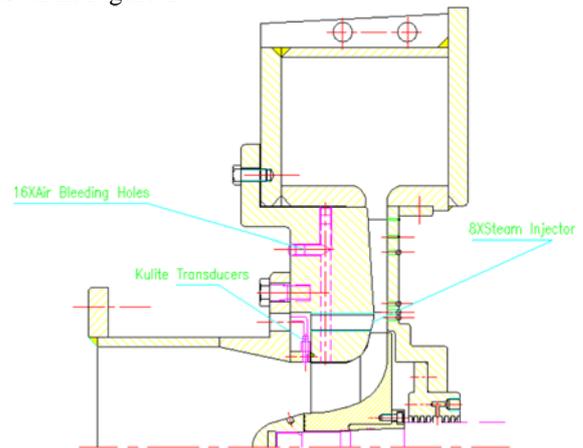
### Test Facilities

The experiments were made on the 500kW DC motor driven test rig at products laboratory of Shaanxi Blower Company. Figure 1 shows a schematic of the test facility.



**Figure 1 Schematic of the Test Facility**

Air at atmospheric condition is drawn into the pipe through an air filter, and through a flow nozzle to measure the flow rate. The settling chamber is used to lower the gas velocity and make the impeller incoming flow more uniform. After the test section, the compressed air is discharged atmosphere. The mass flow rate can be adjusted by two valves. Cross sectional view of the test section is shown in Figure 2.



**Figure 2 Cross Sectional View of Test Section**

The test impeller is scaled down by a factor of 0.5 in order to lower the power consumption to be 4 times smaller so that it can be run by the test rig. For the scaled version, typical design parameters are given in Table 1.

Total Pressure Ratio	-	3.55
Designed Mass Flow Rate	kg/s	3.2
Impeller Tip Speed	m/s	433
Blade Number	-	16+16
Impeller Tip Radius	m	0.115
Average Blade Exit Angle	Deg.	90
Impeller Blade Inlet Angle	Deg.	52
$n_s$ (dimensionless)	-	0.83
Diffuser Inlet Angle	Deg.	1.6
Diffuser Vane Number	-	23
Diffuser Area Ratio	-	2.1

**Table 1 Typical Design Parameters**

In the present test, the compressor was run at 60%, 70%, 80%, and 90% of the design speed to reduce the risk of

potential surge damage at 100% speed. Eight steam injectors were installed midway between impeller tip and diffuser leading edge. Sixteen air bleeding holes formed by radial channel and axial channel were used for impeller throat air bleeding whose results are not covered in current paper.

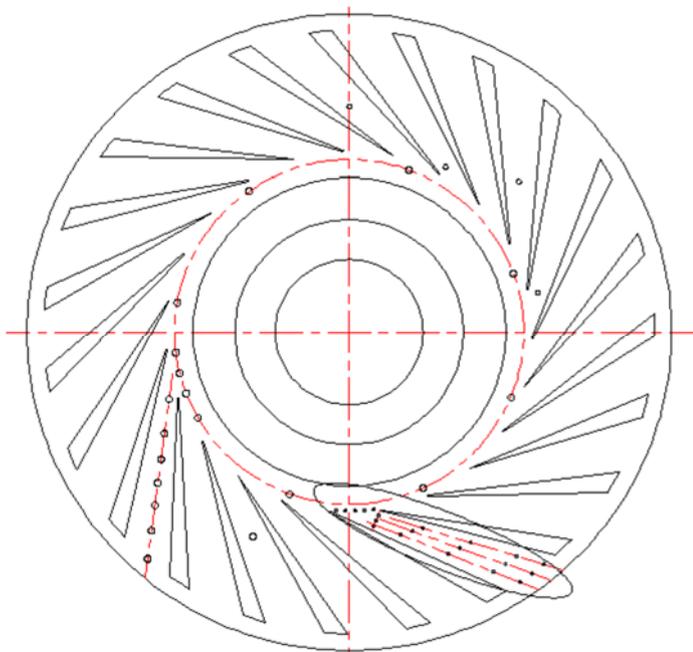
The collector after the diffuser has constant cross sectional area which is chosen to be 4 times of vaned diffuser outlet throat area in order to eliminate its asymmetric influences on upstream components.

**Instrumentation**

Compressor performance was evaluated by measuring the static pressure and temperature at the inlet to impeller and exit of the collector using pressure transducer from Rosemount Engineering Co. A flow nozzle in Figure 1 was used to measure the mass flow rate based on the pressure and temperature of the surroundings. A five-hole cobra probe with a thermocouple was used to measure the total pressure and temperature just at impeller outlet ( $r/r_2=1.04$ ).

For unsteady pressure measurements, 31 Kulite transducers were used. Six of them were uniformly installed 5 mm ahead of impeller leading edge shown in Figure 2. 23 of them were installed at diffuser leading edge, diffuser throat, and inside diffuser channel to record pressure data variation during valve throttling process. The left two were installed on exhaust collector. Besides the 23 unsteady transducers, another 20 steady pressure taps circled by the ellipse shown in Figure 3 were in one channel to investigate pressure contour.

The other two transducers are installed on the collector casing to study the flow behavior inside the collector. A photo of the test rig without shroud is shown in Figure 4 to show the location of the Kulite transducers in the diffuser.



**Figure 3 Steady and Unsteady Pressure Taps in Diffuser**



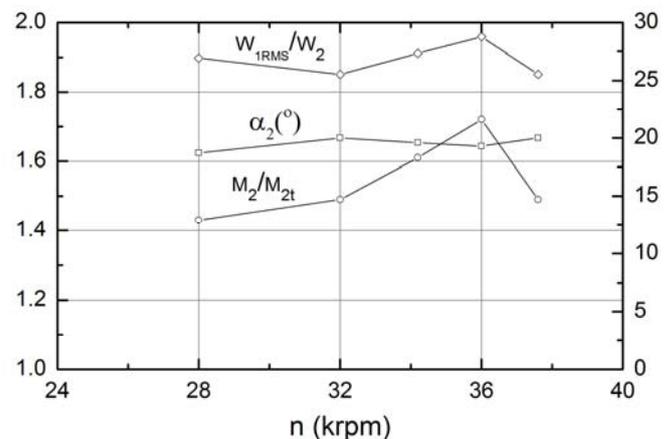
**Figure 4 A Photo of the Test Impeller and Transducers**

During the test, two throttling valves were fully opened for steady operation at each speed line. gradually throttled until the compressor enters into surge. For each speed line, at least six points are measured. At each point, all the unsteady pressure data were acquired and stored for at least 20 seconds. After all six performance data are measured, two valves are opened a little to make it operate at the nearest stable point; then the valves are throttled gradually to push the compressor into surge; after several surge cycles, the valves are quickly opened to 100% in order to pull the compressor out of surge. Unsteady pressure data were acquired all through this process in order to analyze stall/surge precursor and pathology. Sampling frequency of all the 31 transducers was chosen to be 50kHz in order to avoid frequency alias and provide enough time resolution.

**STABILITY ANALYSIS**

Previous studies [1-21] showed that centrifugal impeller/diffuser diffusion limitations could be used as an initial design guideline to indicate the component stalling proximity. Even strongly dependent on personal experiences, they are still widely used in design process.

There are three important compressor surge parameters which are: 1) impeller relative velocity ratio  $W_{1RMS}/W_2$ ; 2) diffuser entry to throat Mach number ratio  $M_2/M_{2t}$ ; 3) impeller outlet absolute flow angle  $\alpha_2$ . The trends of these three parameters along the surge line are plotted on Figure 5.



**Figure 5 Surge Parameters at Different Rotational Speed**

The figure reveals an interesting result in that a line of constant impeller exit air angle almost exactly matches the reported surge line location over the speed range investigated. This could imply that stage surge is dominated by the diffuser vane incidence.

The values of flow angles calculated in Figure 5 seem to be a kind of pessimistic. However, they do reflect the sensitivity of diffuser vanes to incidence. The absolute value is not relevant as its calculation depends on empirical assumptions.

It was however found that the impeller was highly loaded in that surge points average meanline relative velocity ratio  $W_{1rms}/W_2$  was 1.9, which further implies that the impeller could be approaching instability too.

*Suggested Surge Extension Methods*

Based on the above analysis, two methods might be suggested to extend the surge margin of studied compressor. The first method should be able to increase diffuser inlet flow angle or decrease incidence angle. According to the authors' thoughts on using steam from heating recovery boiler, the authors tend to use steam injection at diffuser inlet on the shroud side as the CFD results indicated large mass of low momentum flow accumulation on the shroud side.

To increase the flow angle, the author decided to inject the steam in the reverse direction of impeller rotation which can not only decrease the tangential velocity but also increase the radial velocity. These two factors together increase the diffuser inlet flow angle.

Anyway, this concept is simply based on coarse meanline analysis. If it's effective, the real reason should be explored through detailed numerical simulation and through analysis of pressure signals from 31 installed Kulite transducers.

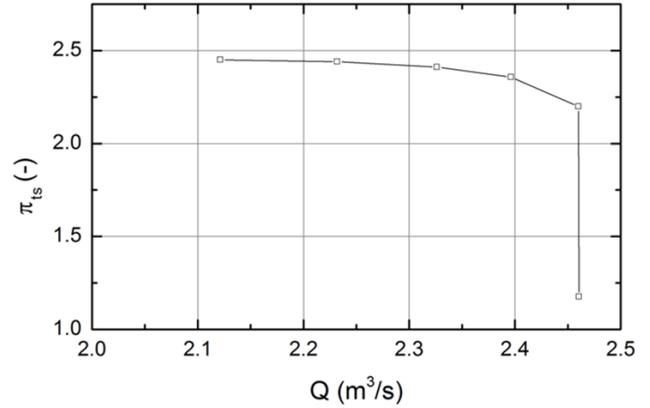
The other way is the notice of impeller is just at the neutral point on the surge line which means the impeller might stall while recovering the diffuser from stall. If the diffuser stopped stall through steam injection, the compressor then can operate at smaller flow rate which increases the impeller loading. And at this point, the impeller loading is increased which may drive the impeller into stall as the impeller stall margin is small as explained before. Therefore, impeller throat air bleeding is suggested as a backup method which has been tested but not included in current report.

*Steady Performance and Unsteady Pressure Data*

For the configuration with vaned diffuser, the measurements were taken at five different speeds.

The rotational speed of the impeller is expressed by a peripheral Mach number  $Mu$  normalized with stage inlet conditions. In this paper, only performance curves at  $Mu$  1.08 are given as an example. The performance map in terms of volume flow rate and total-to-static pressure ratio is shown Figure 6.

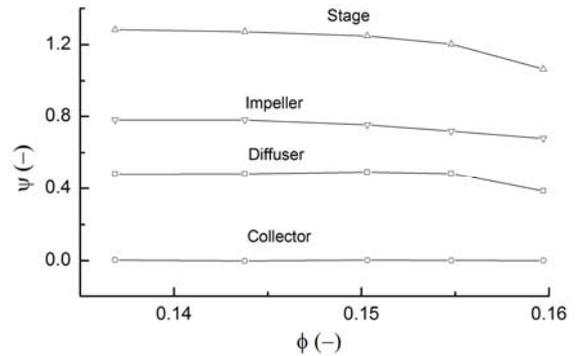
Greitzer's work [1] points out that a stability criterion based on empirical pressure rise versus flow rate data could be deduced. The criterion states that the limit of dynamic stability coincides with the maximum of the total-to-static pressure rise characteristic. Even this is initially applied on axial compressor, it's still shown to be useful in centrifugal one [20]. This argument can also be extended to the individual component of the stage, indicating the components that are potentially unstable [21].



**Figure 6 Stage Pressure Ratio vs Volume Flow Rate**

With the pressure transducers at different positions on the diffuser hub side shown in Figure 3, the performance curves of impeller, vaned diffuser, and collector at  $Mu$  1.08 are shown in Figure 7. The choke point in Figure 6 was removed in order to make the curve slope to be clear due to the steep choke line.

In this figure, the characteristics of both diffuser and collector are taken to be static-to-static which is also adopted in [21]. Also characteristics obtained for each component are given in dimensionless form with stage inlet as the reference position.



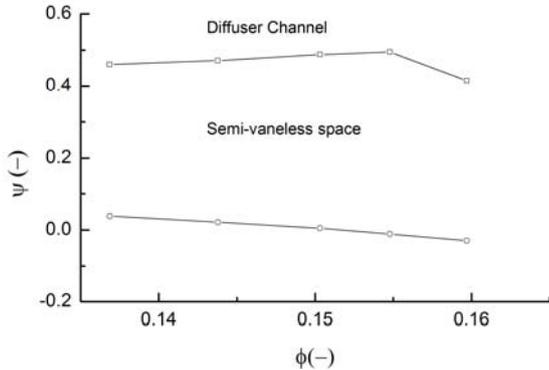
**Figure 7 Characteristics of Components**

From Figure 7, it could be found that the diffuser always has destabilizing effect except for high flow rate; the impeller has a stabilizing effect over all the operating range. The collector is neutral at all flow rates and exerts almost zero static pressure increase which means that the collector has fulfilled its design aim. The almost flat shape of the collector indicates that it has little influence on both performance and stability which is the design aim of this component.

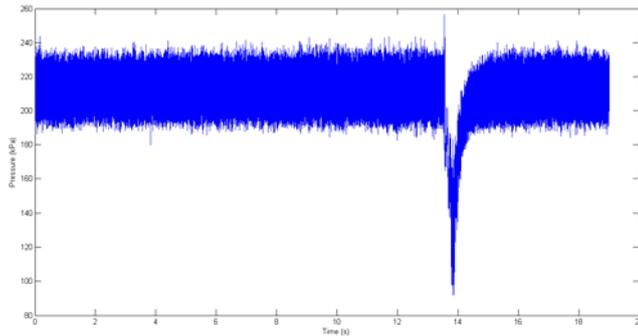
The diffuser destabilizes at most of the left operation range which has positive slope and could be clearly observed in consideration of the diffuser into two subcomponents which are diffuser leading edge to throat (or semi-vaneless space), and throat to exit (diffuser channel) shown in Figure 8.

From Figure 8, it could be observed that the positive slope shown in Figure 7 is actually caused by the diffuser channel in most part of the flow range. The static pressure rise coefficient in the semi-vaneless space grows monotonically with decreasing diffuser mass flow. This could be explained using simple velocity triangles. At high mass flow, the flow accelerates into the throat and the static coefficient is negative;

at low flow rate, the flow decelerates and diffusion occurs between inlet and throat. The same analysis can be applied to diffuser channel: the high boundary layer blockage makes the channel pressure rise to decrease. The diffuser channel is dynamically unstable over most of the mass flow range as seen from the positive slope of the performance curve [7].



**Figure 8 Characteristics of Diffuser Subcomponents**

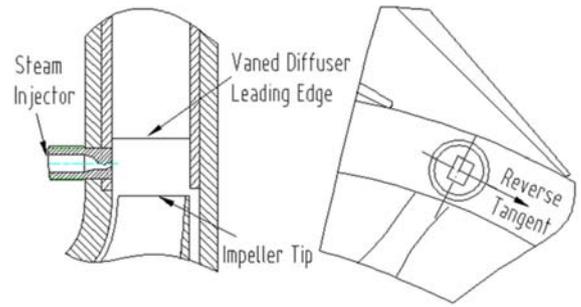


**Figure 9 Pressure Traces at Diffuser Throat**

Since the impeller has a stabilizing effect and the diffuser channel is unstable over most of the flow range, the semi-vaneless space is the subcomponent that determines stage stability which means that the pre-stall activity occurs near the diffuser throat. This can be easily shown by the sudden and first pressure drop approaching surge by transducer at vane diffuser throat. Figure 9 is one of the examples. For all the transducers installed in the compressor, the one at throat is always the first one indicating surge.

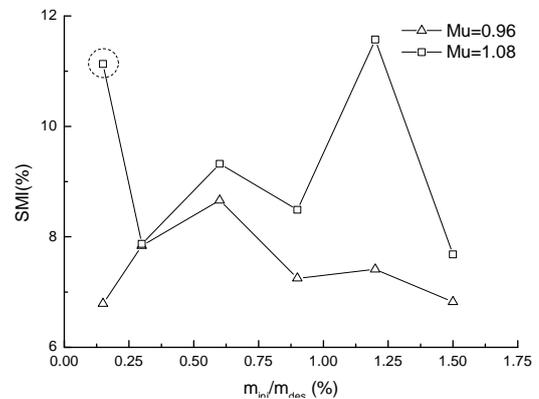
**EFFECTS OF STEAM INJECTION ON PERFORMANCE AND SURGE EXTENSION**

Through the above theoretical and experimental analysis on system stability, it further establishes the confidence of increasing surge margin using steam injection. As the initial procedure and only compressor test (i.e., before testing on gas turbine), the steam is generated from an electric boiler. The maximum steam mass flow rate is 200kg/h. In the current experiment, the steam pressure is kept as 6 bar limited by the used electrical boiler and this number should be higher with HRSG used on site. The saturated steam is injected through eight injectors uniformly installed on shroud side located in the middle of impeller exit and diffuser leading edge to minimize the possible mutual interaction of impeller and diffuser. The position and tip segment of the injector is designed to take advantage of Coanda flow effects [21] as shown in Figure 10.



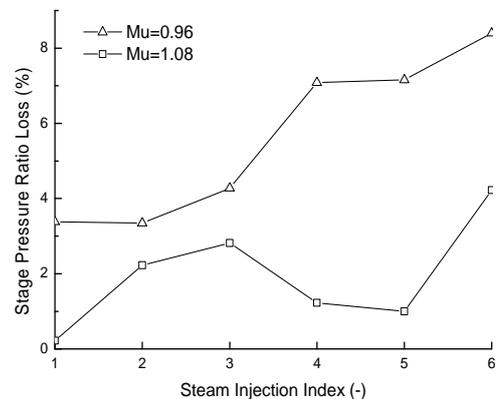
**Figure 10 Steam Injector**

By keeping the impeller rotational speed unchanged, the steam injected mass flow is increased and kept at several different flow rates to investigate its influences on surge extension. At each steam flow rate, valves are gradually throttled to find the surge flow rate. In this paper, surge margin improvement (SMI) is defined as  $\frac{m_{s0} - m_s}{m_{s0}} \times 100\%$ , in which  $m_s$  and  $m_{s0}$  are surge flow rates with and without steam injection respectively.

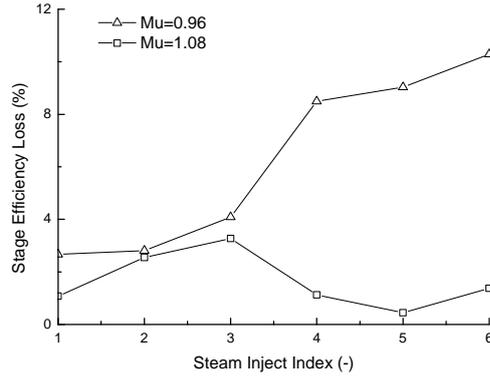


**Figure 11 Steam Injection Percentages on Surge Extension**

Variation of SMI under 80% and 90% of designed rotational speed for six different steam flow rates (I:0.521g/s, II:1.042g/s, III: 2.083g/s, IV: 3.125g/s, V: 4.167g/s, VI: 5.208g/s) are shown in Figure 11.



**Figure 12(a) Stage Pressure Loss vs Steam Injection Index**



**Figure 12(b) Stage Efficiency Loss vs Steam Injection Index**

From Figure 11, it is easy to observe that the trends for these two different rotational speeds are different especially at high steam flow rate. For the line with  $Mu=0.96$ , there is an optimum steam flow for which the surge extension is maximized. This trend is kept as the same for  $Mu=1.08$  except for the first point. For this point only 0.15% designed mass flow rate can increase surge margin over 11%. This effect is the same as the fifth point of which the mass flow rate is seven times as large as that of the first point. The strange phenomenon that the surge margin extension decreased with increasing injected steam mass flow rate. This implies that the traditional explanation based on flow angle might not be correct for the surge inception of centrifugal compressor or at least for this compressor.

After reviewing the test data, it is found that steam injection brings loss to both pressure ratio and efficiency as shown in Figure 12. Stage efficiency is defined by below equation.

$$\eta_{stage} = \frac{T_{t1} \left[ \left( \frac{P_{3,st}}{P_{t1}} \right)^{\frac{n-1}{n}} - 1 \right] m_1 + T_{t1,inject} \left[ \left( \frac{P_{3,st}}{P_{t1,hole}} \right)^{\frac{n-1}{n}} - 1 \right] m_{inject}}{m_1 (T_{3,st} - T_{t1}) + m_{inject} (T_{3,st} - T_{t1,hole})}$$

Even with same steam flow rate, the loss deviates greatly. For example, for the Index equals to 5 (flow rate: 4.167g/s), the loss at  $Mu=0.96$  is almost 8 times that of  $Mu=1.08$ . Another interesting phenomenon is that good SMI points always coincide with good loss ones.

The above phenomena shown by Figure 11 & 12 are repeatable by testing at three different days.

### DISCUSSIONS ON SURGE IMPROVEMENT

Depending upon steam quality (superheated/saturated), nozzle parameter (position/number/angle), and other factors, there are several things that happen with steam injection into the flow path. These things follow:

a. The injection of steam into the flow path has an effect on the properties of the flow, especially in changes to the gas constant, theta, and gamma. This finally changes the operating characteristics of the diffuser.

b. Depending upon temperature difference between steam and air, the heat transfer effect between the two fluids influences the incidence angle of the diffuser through changes in velocity vectors.

c. The different non-uniform temperature in both circumferential and axial direction changes the local corrected speed and local corrected flow which makes different parts of the diffuser work under different inlet flow condition.

d. The steam injection can enhance the flow momentum of the flow near the shroud where wake and separation always exist and delay the flow separation which improves diffuser stability.

e. As the diffuser inlet flow is subsonic, steam injection can also has certain influences on the impeller performance which might also be beneficial for the stability.

### CONCLUSIONS

A centrifugal compressor with vaned diffuser was analyzed in detail through both theoretical and experimental methods. After comparing the steady and analyzing the steam injection results, some conclusions could be made as below,

- Meanline analysis and empirical correlations based on diffusion limitations still show their usefulness in initial design of centrifugal compressor. The semi-vaneless space is shown to be the critical subcomponent for the stability of a vaned diffuser, and the channel or passage after diffuser throat is shown to be dynamically unstable at most flow range. Sizing of diffuser throat should be pay special attention.
- Steam injection is claimed to be an effective method to improve stage stability especially for CCHP gas turbine using steam as power boost medium. Even 0.3% of compressor designed mass flow rate steam can increase the surge margin at  $Mu=1.08$  by over 9.0%. For each speed line, there is an optimum steam flow rate.
- Strange phenomenon shown by steam injection indicates a complicated surge scenario and mechanisms which requires further analysis of acquired mass pressure data.
- Both efficiency and pressure ration drop a little with steam injection.

### ACKNOWLEDGMENTS

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