Influence of Blade Repairs on Compressor Blisk Vibration considering Aerodynamic Damping and Mistuning

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ABSTRACT
Blade-integrated disks (blisks) are used in axial compressors of jet engines due to their lower weight compared to conventional bladed disks. An integrated analysis procedure is presented to calculate the forced response of blisks, incorporating different effects. The influence of random material and geometric variances on the blade eigenfrequencies is calculated first. The aerodynamic coupling between blades is determined using aeroelastic simulations. A Component-Mode-Synthesis based reduced order model of the complete blisk is then used to compute the forced response, considering the aforementioned effects.

The procedure is applied to the critical mode of an axial compressor blisk, and the influence of blend and patch repairs on the resonance amplitude is investigated. Although the procedure assumes cyclic modes with negligible disk participation, which is not the case for the investigated mode, the influence of the repairs on the resonance amplitude was found to be small, suggesting, that the error when comparing them is small as well. More significant impact of the repairs is found for other modes. The dominating effect overall is the aerodynamic damping.

INTRODUCTION
The repair and maintenance of jet engine components involves a considerable part of the aviation industry. Around 8% of the total operating cost of a civil airplane are commonly related to maintenance and overhaul of the engines (Rupp, 2001). Therefore, there are various efforts to improve the maintenance process and develop profitable repair concepts, in particular, for rotating blades (Aschenbruck et al., 2014).

Recently, a trend towards more compact jet engines with a fewer number of components can be observed. For the purpose of a weight reduced design the conventional assembly of the rotor blades and the disks is replaced by a single component, the blisk. The use of innovative blisk technology results in even larger spare parts costs which finally leads to a growing interest in advanced maintenance processes.

The most frequently applied blade repairs involve the blending and patching of the airfoil. Minor defects are removed by blending. Larger damaged areas of the blade can be rebuilt using an additional patch. The high-tech repairs generally permit the continued use of the component within the jet engine, but also leads to a refurbished blisks with modified vibrational properties. Changes in structural
behavior and aerodynamic damping may affect the vibration of blades and may be crucial for a safe operation. Numerical simulations allow the detailed prediction of the blade’s behavior. An extensive study on blade vibrations in turbomachinery engines was conducted by Hou and Wicks (2002), who considered stiffening effects and softening due to temperature. Further investigations on turbomachinery design focused increasingly on the stochastic analysis (Vogeler and Voigt, 2015). Uncertainties in material data and geometrical variances are considered to cover the real structural properties of blades. The impact of scattering properties due to manufacturing tolerances as well as due to repair-induced imperfections is evaluated by (Berger et al, 2016a and 2016b).

A major challenge in predicting the vibrational response of mistuned structures is the consideration of each individual component. In order to keep the computational cost low, reduced order models (ROM) are applied for simulation. Reduced order analyses including aerodynamic damping of a tuned transonic fan by Mårtensson et al. (2008) show an aerostatically stable coupling between mode families in a narrow frequency range. The Multimode Least Square (MLS) method by Mayorca et al. (2012) considers the aerodynamic mode interaction in a Guyan-reduced approach.

The influence of mistuning on the vibration response of bladed disk is studied in numerous publications. A good overview can be found in (Castanier and Pierre, 2006). Mistuning describes the loss of symmetry in bladed disks, which manifests itself in differences of the structural properties of individual blades. It leads to a localization of vibration energy and therefore increased vibration amplitudes of one or more blades. The amplitude is very sensitive to small deviations of the structural properties, especially in systems with low damping.

Because the cyclic symmetry of mistuned bladed disks is lost, it is not possible to account for mistuning effects in a sector model with cyclic boundary conditions, so ROMs are used extensively. A recent review of different model reduction techniques can be found in (Nyssen, 2016). Component Mode Synthesis based methods allow to analyze arbitrarily differing blades on a disk. The SNM method (Subset of Nominal Modes) is only valid for small deviations from the tuned structure (Yang and Griffin, 2001), whereas the popular CMM method (Lim et al., 2007) is limited to blades with the same number of dof. Techniques based on the modified modal domain analysis (Sinha, 2007) can account for varying blade geometry.

OUTLINE

The remaining sections are organized in the following way: First the analysis procedure to assess the influence of blade repairs is described. An overview over the whole procedure is given. The procedure consists of three types of analyses, which are detailed after the overview. The section “Application and results” deals with the application of the analysis procedure to a compressor blisk with different blade modifications. As before, a general overview is followed by the individual results of the three analyses. Finally a conclusion is given.

ANALYSIS PROCEDURE

The analysis procedure to assess the influence of the repairs, as shown schematically in Figure 1, consists of three parts:

First, stochastic structural analyses are performed for nominal and repaired blades using FEM. The simulations result in the distributions of the blade’s eigenfrequencies for each repair, considering additional material and geometric deviations.

Second, an aeroelastic analysis is performed with one-way fluid-structure-interaction. For each repair and the nominal geometry the stationary flow is calculated assuming cyclic symmetry using a RANS solver. The CFD-mesh is then deformed according to the structural mode shapes and equivalent damping and stiffness contributions of the fluid are calculated from the resulting pressure variation on the blade surface.

Third, the whole blisk is considered using a reduced-order-model based on the Component-Mode-Synthesis with Wave-Based-Substructuring. The aerodynamic coupling of the blades is incorporated and Monte-Carlo-Simulations are performed using the blade eigenfrequency distribution from the stochastic structural analysis. The probability distribution of vibration amplitudes at resonance is thereby determined for the nominal operating point.

![Figure 1 Schematic representation of analysis procedure](image)

Stochastic Modal Analysis

The vibrational properties of the single blades are evaluated numerically solving the following equation:

\[
(-\mathbf{M}_{\text{blade}}\omega^2+\mathbf{K}_{\text{blade}})\Phi = 0,
\]

where \(\mathbf{M}_{\text{blade}}\) is the mass matrix, \(\mathbf{K}_{\text{blade}}\) the stiffness matrix, \(\omega\) the eigenvalues and \(\Phi\) the matrix of eigenvectors. Stress stiffening effects which are caused by the rotational speed during the operation of the engine are considered. Therefore the stiffness matrix is modified by an initial stress term according to the operational conditions.

Furthermore, the scatter in material properties is involved in the modal analysis. Four normal distributed material parameters specified in the following sections

\[
\implies
\]
(Table 2) form the basis for the stochastic evaluation of eigenfrequencies and modes using Latin Hypercube Sampling. Thus, a detailed description of the scattering eigenfrequencies of the single blade is provided.

Aeroelastic Analysis

The simulation toolchain employed in the aeroelastic analysis has been developed at the DLR and is described in several papers (e.g. Kersken et al., 2012). It uses the solver TRACE for the compressible Reynolds averaged Navier Stokes (RANS) equations, a mapping algorithm that maps structural modes from a finite element solution to the blade surface of the finite volume mesh, a mesh deformation algorithm and a time-linearized solver, which performs the aeroelasticity calculations. Keller et al. (2017) describe the solution process in more detail.

The stability of the blade is evaluated by calculating the modal aerodynamic work per cycle performed on the blade. The aerodynamic work is the product of the unsteady pressure fluctuations (solved for using the time-linearized solver) and the blade displacement vectors in each cell on the blade surface. If the real part of the aerodynamic work is positive, meaning that energy is transferred from the fluid to the blade, the system is aerodynamically unstable. Finally, an aerodynamic damping coefficient can be computed from the modal work per cycle under the assumption of viscous damping. A concise form of the damping modelling equations is presented by Willeke et al. (2017). A short summary is given here.

The matrix $K_a$ in Eq. (3) (explained below) is made up of aerodynamic influence coefficients, which express the aerodynamic stiffening and damping influence between each blade and each mode. These complex coefficients are computed from the complex aerodynamic work

$$W_{t,j} = -i\pi \Phi_j H_i$$

using

$$k_{a,i,j} = -\frac{1}{\pi \omega A^2} \left(\text{Re}(W_{t,j}) - i\omega \text{Im}(W_{t,j})\right)$$

with the frequency of vibration $\omega$ and the modal amplitude $A$ with respect to complex eigenvectors $\Phi$ normalized to a modal mass of 1. Vector $\hat{f}$ contains the complex amplitudes of the local aerodynamic forces on the blade.

For the following mistuning analysis, only the modes near the frequency of interest are needed. Since it is possible, that the excited blade vibration consists of two or more modes, their interaction has to be considered in the aeroelastic analysis as well. Because small amplitudes are assumed and the pressure fluctuations can be computed using the linearized Navier-Stokes Equations, cross modal aerodynamic work can be calculated in a simplified manner and later be used in a superposition of the real mode shape. The aeroelastic simulations are only performed for the excited blade vibration consisting of two or more modes.

The aeroelastic analysis is performed for all possible nodal diameters of the blisk. In contrast to this, the mistuning analysis requires stiffness coefficients to represent the coupling of the structure via the flow. By applying a Fourier transform to the complex aerodynamic stiffness in eq. (3), the traveling wave representation can be transformed to an influence coefficient representation useful for the structural reduced order model.

Mistuning Analysis

To model the blisk and account for aerodynamic effects and structural mistuning, a ROM based on the Component Mode Synthesis is used. The original method is presented in (Hohl et al., 2009), and the implementation of aerodynamic coupling between blades is detailed in (Willeke et al., 2017). The procedure is modified slightly, as described in the following description.

The bladed disk is divided into three parts, which are reduced differently. The blades and the disk are reduced using the Craig-Bampton reduction, with the common (boundary) dofs of the disk and the blades being reduced additionally using Wave-Based-Substructuring (WBS, Donders, 2006).

To reduce the boundary dofs, cyclic modal analysis of the whole blisk is performed for each nodal diameter. Only the displacements of the boundary dofs are kept for each mode and the singular value decomposition is performed according to the WBS method individually for each nodal diameter, yielding the “Waves” $W_j$ for nodal diameter $j$. The Blades and disk are then reduced with the well-known Craig-Bampton reduction (Bampton and Craig, 1968), using the Waves as interface modes. Cyclic symmetry is exploited for the disk and the boundary dof. For each nodal diameter, the interior dof $x_i$ and the boundary dof $x_j$ of the disk are transformed in the following way:

$$\begin{bmatrix} x_{i,j} \\ x_{b,j} \end{bmatrix} = \begin{bmatrix} \Phi_j & \Psi_j W_j \\ 0 & W_j \end{bmatrix} \begin{bmatrix} \xi_{i,j} \\ \xi_{b,j} \end{bmatrix}$$

The reduced interior and boundary dof are $\xi_i$ and $\xi_b$ respectively, and the fixed interface modes and constraint modes are denoted by $\Phi$ and $\Psi$. For the blades, the approach is similar. The whole reduction procedure is performed for the reference geometry only. Then the results of the other analyses are incorporated: The aerodynamic work for each blade mode is used to calculate equivalent modal damping and stiffness contributions $D_r$ and $K_r$. This is carried out for each blade modification without additional mistuning. The blades stiffness matrices are multiplied by the factors $\delta_j$ to implement the frequency mistuning:

$$K_{\text{blade},j} = (1 + \delta_j)K_{\text{blade},\text{tuned}}$$

The distribution of these random factors is chosen based on the expected value and standard deviation of the stochastic modal analysis, thereby incorporating the individual blade repair in addition to random material and geometric variations.

The forcing $\hat{f}$ is assumed to be harmonic with an amplitude of 5% of the nominal static pressure forces on the blade.
surface, with the engine order corresponding to the number of upstream stator blades. This leads to the following linear equation of motion in the frequency domain:

\[-\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}_a + (1 + i\omega)\mathbf{K}_{\text{mismatched}} + \mathbf{K}_d\mathbf{x} = f\]  

(5)

The overall analysis procedure enables the simulation of blisks, yielding the forced response and probability based data about the amplitude increase due to mistuning. Aeroelastic effects, and the influence of numerous uncertainties are modelled. The procedure is computationally efficient, because only the blade substructure is considered in the stochastic modal analysis of the full model and during the aeroelastic simulations. As a consequence of this restriction, the results of the stochastic modal analysis and aeroelastic analysis do not account for movement of the blade root. The error introduced by this is dependent on the amount of disk participation in the cyclic mode analysed. Therefore, the procedure is best used for blade dominated modes.

APPLICATION AND RESULTS

The method is applied to the compressor blisk (blade integrated disk) of a single-stage high-speed axial compressor with inlet guide vane of the Institute of Turbomachinery and Fluid Dynamics at Leibniz University Hanover. The configuration was designed and built during the course of a four year research project, which is part of the Collaborative Research Center 871 “Regeneration of Complex Capital Goods” (Keller et al., 2015). The geometry of one blisk sector is shown in Figure 8a. Additional general information is given in Table 1, in comparison with a four-stage configuration of the test rig.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>4-stages</th>
<th>CRC 871</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational Speed</td>
<td>1/min</td>
<td>18,000</td>
<td>17,100</td>
</tr>
<tr>
<td>Power Input</td>
<td>kW</td>
<td>950</td>
<td>300</td>
</tr>
<tr>
<td>Tip Diameter</td>
<td>mm</td>
<td>340</td>
<td>340</td>
</tr>
<tr>
<td>Pressure Ratio</td>
<td>-</td>
<td>2.95</td>
<td>1.37</td>
</tr>
<tr>
<td>Mean Stage Pressure Ratio</td>
<td>-</td>
<td>1.31</td>
<td>1.37</td>
</tr>
<tr>
<td>Isentropic Efficiency</td>
<td>-</td>
<td>0.88</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The axial compressor has been specifically designed to investigate the influence of regeneration on the aeroelastic stability and forced response of a compressor blisk. Due to its low mechanical damping, the interaction with the flow becomes a dominant damping mechanism. Since blade repairs, such as blend repairs at the leading edge, have a significant impact on the mean flow, they also cause a change in aerodynamic damping. Depending on the position of the repair, this change may be quite significant (Keller et al., 2017).

Figure 2 Modified blades a), b) and c) with blending and d) patch at the leading edge

The blade repairs investigated in this paper are shown in Figure 2. Figures 2.a, 2.b and 2.c show three different blend repairs at 60% span, 80% span and the blade tip while the fourth modification (Fig. 2.d) is a patch repair at the leading edge. The last modification has the same geometry as the reference blade, but different material properties in the patch region. The modifications were applied to all blades simultaneously. E.g. a blisk with modification c) has 24 blades, each with an identical blend repair at the tip. This is necessary to allow periodic boundary conditions in the aeroelastic analysis.

Figure 3 shows the interference diagram for the blisk without modification at nominal rotational speed. The excitation frequency corresponding to Engine order 23 excitation is drawn, matching the number of upstream stator blades. The mode closest to resonance during nominal operation matching EO 23 is highlighted. It has one nodal diameter and belongs to the sixth mode family. This mode will be the focus of further investigations. To enable excitation of this mode at resonance, the analyses are performed at a slightly reduced operating speed of 16,480rpm.

Figure 3 Interference diagram of the blisk at operating speed with the critical mode highlighted

Stochastic Modal Analysis

For the modal analysis five different finite element models are used in order to describe the blade’s
modifications sufficiently accurate. The models all include one single airfoil, but not any further part of the blisk. The airfoils are assumed to be firmly clamped and the displacements in axial, tangential and radial direction are constraint at the bottom of the airfoil. Tetrahedral and hexahedral elements are used within the model according to the complex geometry of the blade and its modifications. Each finite element mesh of the reference or modified blades comprises at least 70,000 nodes. The blisk is made of titanium alloy Ti-6Al-4V and the related material parameters are assumed to be normal distributed with the statistical parameters given in Table 2.

### Table 2 Scattering material properties according to (MIL-HDBK-5J, 2003)

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Unit</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>GPa</td>
<td>116.52</td>
<td>0.2</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>-</td>
<td>0.31</td>
<td>0.02</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>4430</td>
<td>13.3</td>
</tr>
<tr>
<td>Thermal extension coefficient</td>
<td>1/K</td>
<td>8.82e-6</td>
<td>0.2e-6</td>
</tr>
</tbody>
</table>

By means of Latin Hypercube Sampling, for each modification 1000 realizations are generated and computed. The first six eigenfrequencies are analysed, because they describe the vibration characteristics of the blade sufficiently accurately.

The distribution of the first eigenfrequency related to the reference and one modification, which covers the frequency range of 10 Hz, is shown in Figure 4a. In comparison to the reference model the blending leads to a slight shift towards higher frequencies. The main reason for the frequency deviation is the lower mass of the refurbished airfoil. Particularly, the first bending mode (Figure 4b) illustrates, that the removal of material next to the tip of the blade induces higher frequencies. The same effect could be found for the other two blending repairs (a) and b).

But the reduced mass does not generally lead to higher eigenfrequencies. The histogram of the second eigenfrequency corresponding to the blending modifications (a-c) in Figure 5 shows, that in some case the mean values of frequencies obtained by modal analysis are smaller than the frequencies of the reference model. The different influences of blending repairs can be explained by the position of the blending and the related mode shape (Figure 5b). In contrast to modifications b) and c) the modification a) leads to a shift towards lower frequencies. In this case the position of the blending coincides with the vibration node and therefore the influence of a reduced mass on the eigenfrequency is negligible and the shift in eigenfrequency results from reduced stiffness properties.

### Table 3 Mean and standard deviation of the 6th eigenfrequency due to scattering properties

<table>
<thead>
<tr>
<th>Modification</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>7239.5</td>
<td>13.40</td>
</tr>
<tr>
<td>Modification a)</td>
<td>7220.1</td>
<td>13.48</td>
</tr>
<tr>
<td>Modification b)</td>
<td>7252.0</td>
<td>13.64</td>
</tr>
<tr>
<td>Modification c)</td>
<td>7239.3</td>
<td>13.26</td>
</tr>
</tbody>
</table>

**Aeroelastic Analysis**

Aerodynamic damping has been computed for an excitation frequency of roughly 6317, which represents the excitation order 23 at a rotor speed of 16,480 rpm. Since only modes 5 and 6 have significant share in the resulting shape of the forced vibration, the analysis was limited to these modes.
(aerodynamic damping of all other modes was set to zero). Some excerpts of the results are shown in Figures 6 and 7.

Figure 6 Aerodynamic damping for displacement of mode 5 and pressure field of mode 5

Damping curves have a somewhat sinusoidal shape in most cases. The presented results show no exception to this general observation. Figure 6 shows the aerodynamic damping for the interaction of mode 5 with its own pressure oscillation field. The first and second repair differs only slightly in average damping and amplitude of the curve. In the influence coefficient representation, the average value of the damping curve represents the influence of a blade on itself, the first harmonics in a Fourier transformation represent the influence on the adjacent blades and so on. Note that this explanation is simplified, because the complex values according to eq. (2) should in fact be used. Repair c) not only shows a significant shift in average damping value but also in the amplitude of the curve, meaning that the damping of the influence of the blade on itself and the influence of the directly adjacent blades is significantly different. The explanation for this behavior is the fact, that changes in mean flow have a much higher impact on damping behavior than the small changes in mode shape due to the repair. Thus, geometry and therefore flow changes in the tip region have a significantly higher impact on the aerodynamic work performed than changes in geometry closer to the hub. The same behavior has previously been observed and discussed in more detail by Keller et al. (2017).

Figure 7 Aerodynamic damping for displacement of mode 5 and pressure field of mode 6

Figure 7 shows the aerodynamic damping for the interaction of mode 6 with the pressure field of mode 5. Apart from very similar overall trends, another observation can be made. For several inter blade phase angles, the damping coefficient becomes negative, possibly leading to an unstable system. For all other interactions aerodynamic damping is positive so that overall no instability is to be expected. The aerodynamic damping coefficients for mode 6 are not shown here, because similar trends were found between the modifications a)-c), resulting in small differences in damping. The aerodynamic damping of this mode is smaller than the damping of lower frequency mode families, especially at the nodal diameter investigated here.

Mistuning

The finite element mesh of a blisk sector with reference geometry, used as basis for the reduced order model in the mistuning analysis is depicted in Figure 8a. It consists of 25148 nodes and mostly Hex20 elements.

Structural (hysteresis) damping of d = 0.28% is assumed for the titanium alloy, approximating the values measured in (Lee et al., 2008). Because the authors did not measure the damping in a vacuum, the actual damping is lower, and the relative influence of the aerodynamic damping will be greater in reality, compared to the results of this study. Monoharmonic excitation with engine order 23 is used. For the reduction, the first 20 blade modes, and 5 disk modes and 5 waves where used for each nodal diameter. Due to the high number of modes included, the final reduced model is of moderate size, with 720 degrees of freedom. All modifications can be represented by stiffness variations of the reference blade, as shown in eq. (6), because of the small amount of mistuning due to the blade modifications.

The mode shape of the critical mode analysed is shown in Figure 8b. Although somewhat similar, it differs considerably from the blade-alone modeshape used in the preceding analyses. Nonetheless, some conclusion can be drawn from the application of the analysis procedure.
Furthermore, it is planned to improve the overall procedure in the future, by extending the individual analyses and coupling them more tightly. By comparing the results the influence of the current simplifications can then be judged.

Figure 9 shows the calculated frequency response of the tuned blisk for the different blend repairs. Without the influence of aeroelastic effects, a small frequency shift due to the repairs is noticeable. The aeroelastic damping reduces the amplitudes considerably, while the negative equivalent aerodynamic stiffness reduces the eigenfrequencies. The aerodynamic effects are very similar for all blade variants for the analysed mode, as expected.

To analyse the influence of random mistuning, Monte-Carlo simulations are performed for modifications a), b) and c) with different amounts of random blade stiffness mistuning. For each mistuning amount 500 realizations were simulated. The mistuning is modelled with a Gaussian distribution, using the results from the stochastic modal analysis for each repair. The same distribution is sampled independently for all blades. Figure 10 shows the results. Here, the amplitude magnification factor due to mistuning is plotted against the amount of mistuning, which is given as a percentage of the stochastic results. Because the overall damping and the eigenfrequency standard deviation for the modifications are very similar, little difference in amplitude magnification is found. The overall amplitude increase caused by random mistuning is low, because the amount of mistuning is small, considering the amount of structural and aeroelastic damping. Even though this part of the analysis procedure is not expected to be accurate for the mode investigated, the results are plausible and match the expected behaviour, given the results of the other parts of the analysis procedure, namely little difference in aerodynamic damping and random eigenfrequency deviations. It is planned to reexamine this problem again using a refined analysis procedure, to confirm this assumption.

CONCLUSIONS

A comprehensive analysis procedure is presented to estimate the forced response of mistuned blisks with varying geometry due to repairs, considering aeroelastic effects and multiple sources of mistuning. The procedure is divided in three steps, which can be performed in order: A stochastic modal analysis, yielding a distribution of blade-alone eigenfrequencies depending on varying geometric and material properties. Aeroelastic simulations result in an equivalent damping and stiffness of the fluid for each mode. These data are then used as input for the mistuning analysis, which uses a Component Mode Synthesis based reduced order model. The procedure is valid only for blade-dominated modes, but is shown to produce plausible results even in the application shown here. The validity of the results is subject to further investigation using a refined analysis procedure. Using the procedure as presented here, little influence of the analysed blisk repairs and random mistuning on the vibration amplitude was found. The two most influential effects are the aeroelastic damping and the change in eigenfrequencies, while the effect of mistuning is similar for all variants. The distance between resonance and excitation during nominal operation can be computed easily using regular modal analysis. The detailed modelling of aeroelastic effects should therefore be the focus of further efforts.

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REFERENCES


