Numerical Study of Buoyancy-Driven Flow in a Closed Rotating Annulus

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ABSTRACT
This paper presents a numerical investigation of buoyancy-driven flow in a closed rapidly rotating disc cavity. Pseudo two-dimensional models are considered, with periodic boundary conditions on a thin axial domain. An incompressible model, in which density variation is considered with the Boussinesq approximation, is evaluated through comparisons with a full compressible model. Effects of property (viscosity) variation and dependency on buoyancy parameter ($\beta \Delta T$) and rotational Reynolds number for a given Rayleigh number, are investigated with the full compressible model. The mean centrifugal and radial Coriolis forces are analysed. Heat transfer predictions from the Boussinesq and compressible models agree to within 10%, for $\beta \Delta T \leq 0.2$.

INTRODUCTION
Economic and ecologic needs have driven manufacturers to design lower fuel consumption jet engines, by increasing by-pass ratios and overall pressure ratios. This requires smaller engine cores, and results in a proportionally larger compressor rotor blade tip clearance ratio, which must be controlled during different operating conditions across the engine cycle. The tip clearance of the compressor rotor blade is strongly dependent on the thermal growth of the compressor rotor discs, and in turn the heat transfer through the discs and the shroud. Efficient control of disc thermal growth requires clearer understanding on the mechanisms and reliable prediction tools of the flow within the rotor drum cavity.

An example of a compressor rotor disc cavity is given in Figure 1, reproduced from Fitzpatrick (2014). In this figure, open rotating cavities are formed between a shroud, discs and cobs. Axial throughflow, at the bore between the shaft and disc cobs, expected to cool the discs and shrouds which conduct heat generated by compression of gas in the main annulus flow. In some configurations radial inflow through the shroud may be used to cool the discs, or sealed cavities with no throughflow may occur.

The flow in rotating compressor cavities without radial throughflow is known to be buoyancy-driven, and usually has long time scale, i.e. it takes long time (compared to forced convection flows) to get to a steady state from a previous operating condition. This feature of the flow challenges both numerical predictions and experimental measurements. The understanding of the mechanisms of such flows has been reviewed by Owen and Long (2015). Some progress in understanding has been achieved, but researchers are still struggling in accurately simulating the buoyancy-driven flow inside the rotating cavities.

Recently, Pitz et al. (2019) reported, for the first time, statistics of the thermal and kinematic boundary layers in a sealed rotating cavity with heated shroud and cooled shaft. These were obtained from large-eddy simulation (LES). Inconclusive results have been obtained in comparing
predicted and measured shroud heat transfer. Pitz et al.’s (2019) predicted shroud Nusselt number agreed with the correlation for heat transfer between horizontal plates under gravity but not Bohn et al.’s (1995) results for a rotating cavity. However, Pitz et al. (2019) used an incompressible model and modelled the effect of density variation with Boussinesq approximation. This approach is well accepted for investigations of Rayleigh-Bénard convection under gravity but has not been validated for centrifugal buoyancy problems, in which centripetal acceleration is an additional factor.

The model considered by Pitz et al. (2019) is based on the experimental rig from Aachen University (Bohn et al., 1995). The definition of the Rayleigh number \( Ra \) (as used by Bohn et al.) is given in Eq. (1), where \( Pr \) is Prandtl number, \( \beta \Delta T \) is the buoyancy parameter, \( Re_\phi \) is the rotational Reynolds number, and \( a \) and \( b \) are the radii of the shaft and shroud. Further notation is defined in Figure 2 and the Nomenclature section at the end of the paper. For a given \( Ra \) there are infinite possible combinations of \( \beta \Delta T \) and \( Re_\phi \). The rig operating condition was defined by a relation given in Eq. (2) for the configuration considered here, and a correlation was derived between the measured shroud Nusselt number (\( Nu \)) and \( Ra \). However, the separate effect from \( \beta \Delta T \) and \( Re_\phi \) on shroud \( Nu \) is not clear.

\[
Ra = 2Pr\beta\Delta TRe_\phi^{\frac{b-a}{b+a}} \quad (1)
\]

\[
Re_\phi = 1.441Ra^{0.557} \quad (2)
\]

**Figure 2 Illustration of Aachen University’s Rig, Configuration B (Bohn et al., 1995)**

The main aims of this paper are as follows.

(1) to investigate the validity of incompressible models with the Boussinesq approximation for centrifugal buoyancy-induced flow in a rapidly rotating disc cavity through comparisons with full compressible models.

(2) to understand the separate effect from \( \beta \Delta T \) and \( Re_\phi \), and the effect of property variation (e.g. viscosity \( \mu \)) on shroud \( Nu \).

**CONFIGURATION STUDIED**

The geometry considered in this paper is configuration B of Bohn et al.’s (1995) rig, and as studied by Pitz et al. (2019). A schematic illustration of the geometry is given in Figure 2. This configuration is an air-filled closed annulus with geometric parameters \( a, b \) and \( d \) being 0.125m, 0.24m and 0.12m, respectively. The shroud is heated and the shaft is cooled. The two discs are considered as adiabatic. Shroud \( Nu \) were measured following the operating conditions defined by Eq. (2). A correlation between the measured shroud \( Nu \) and \( Ra \) was obtained, as given in Eq. (3). Later, Bohn and Gier (1998) noted that the discs were not perfectly adiabatic, and subsequently introduced a corrected version for the \( Nu - Ra \) correlation, given in Eq. (4).

\[
Nu = 0.317Ra^{0.211} \quad (3)
\]

\[
Nu_{corr} = 0.0677Ra^{0.297} \quad (4)
\]

In this paper the configuration in Figure 2 is further simplified to a pseudo 2D test case, by shortening the axial extent of the model to \( d = 10^{-3} \)m and substituting no-slip discs with periodic boundary conditions. This very significantly reduced the computing requirements for the simulation. The 2D assumption not only removes any effect of the disc end walls, but restricts the turbulent motion. Although similar 2D assumptions have been used by others (King et al., 2005), some uncertainty regarding its effect must be acknowledged.

**Test Matrix**

The test matrix used for this study is summarised in Table 1. Parameters given in this table also include the Eckert number \( Ec = \Omega^2b^2/(Cp\Delta T) \). This is sufficiently small for the use of temperature transport equation as energy equation in the incompressible model to be considered appropriate. As well as rig condition tests, the matrix includes variation of \( \beta \Delta T \) for \( Ra = 3.3 \times 10^9 \) and a fixed viscosity \( \mu \) in the compressible model. Note that the same symbols as seen in Table 1 will be used in later plots. For all the tests statistics are collected over at least 50 rotor revolutions after a statistically steady state is reached. All the tests are run with direct numerical simulation (DNS), except the condition \( Ra = 10^9 \) for which Hydra employs large-eddy simulation (LES).

<table>
<thead>
<tr>
<th>Test Matrix</th>
<th>Bohn et al.’s rig condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ra )</td>
<td>( 10^7 )</td>
</tr>
<tr>
<td>( \beta \Delta T )</td>
<td>0.174</td>
</tr>
<tr>
<td>( 10^{-4}Re_\phi )</td>
<td>1.14</td>
</tr>
<tr>
<td>( 10^2Ec )</td>
<td>0.0723</td>
</tr>
<tr>
<td>Semtex</td>
<td>( \triangle )</td>
</tr>
<tr>
<td>Hydra</td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td>Hydra fixed ( \mu )</td>
<td>( \bigcirc )</td>
</tr>
</tbody>
</table>

**METHODOLOGY**

The Navier-Stokes (N-S) equations are assumed to govern the problem described in this study. Numerical solutions were obtained using two codes, Hydra and Semtex.
Hydra

Hydra, owned by Rolls-Royce plc, is a full compressible N-S solver based on finite volume method with node-vertex unstructured meshes. A recent development (Amirante and Hills, 2015) extended Hydra to second order accuracy through linear reconstruction of primitive variables for flux estimation. The monotone upwind schemes for conservation laws based on a modified Roe scheme is used for spatial discretisation. The time interpolation employs an explicit 3-stage Runge-Kutta scheme. The working fluid is assumed to be air modelled as a perfect gas. The dynamic viscosity is calculated according to Sutherland’s law.

Semtex

Semtex is an opensource spectral-element-Fourier code solving the incompressible form of the N-S equations (Blackburn and Sherwin, 2004). The finite element concept is applied on a basic 2D geometry and high-order Lagrange polynomials are implemented within each parametrically mapped quadrilateral element. Fourier expansions are applied to the third direction, which has to be homogenous. A semi-implicit second-order scheme (Karniadakis et al., 1991) is used for time discretisation.

In order to model centrifugal buoyancy-driven flow, the effect of density variation \( \rho' = \rho - \rho_0 \) must be considered in the centrifugal force term in the momentum equation as given below in Eq. (5).

\[
\rho_0 \frac{\partial \bar{u}}{\partial t} + \rho_0 \bar{u} \cdot \nabla \bar{u} = -\nabla p + \rho_0 \nabla^2 \bar{u} - 2\rho_0 \bar{\Omega} \times \bar{u} - \rho \bar{\Omega} \times (\bar{\Omega} \times \bar{r})
\]

where \( \bar{r} \) is the centrifugal buoyancy term and \( \bar{\Omega} \) is the reduced pressure,

\[
P = p + 0.5\rho_0 (\bar{\Omega} \times \bar{r})^2
\]

With the Boussinesq approximation, the density variation is neglected in all other terms. Density variation in the centrifugal buoyancy term is approximated, with

\[
\rho' = -\beta(T - T_0)
\]

where \( \beta = 1/T_e \), for a perfect gas is the thermal expansion coefficient. The pressure is obtained by solving the Poisson equation, and the transport equation of temperature is considered as the energy equation.

Here we regard the N-S equation form as the primary difference between the two models. More details about the two solvers are given by (Amirante and Hills, 2015) and (Blackburn and Sherwin, 2004).

RESULTS AND DISCUSSION

Effect of Mesh Resolution

Mesh resolution studies have been conducted for all the operating conditions considered in this paper. An example is given for the Hydra solution with \( Ra = 10^8 \). Table 2 shows the effect of mesh resolution on shroud \( Nu \) and averaged core temperature. Figure 3 shows how the mesh resolution affects the flow field. At non-dimensional time \( t = \tau \), the fine mesh solution is interpolated to the coarse mesh and solutions are continued on both meshes. After two rotor revolutions (at \( t = \tau + 4\pi \)) the “plumes” in the coarse mesh solution are more diffused, and under-prediction of the shroud \( Nu \) is seen in Table 2. The grid resolutions used in all subsequent results in this paper are given in Table 3. In the axial direction three mesh nodes are used in Hydra, and the axial plane number in Semtex is polynomial order plus one.

**Table 2: Effect of Mesh Resolution on Hydra Solution for \( Ra = 10^8 \) (Rotor Revolution Time=2\pi)**

<table>
<thead>
<tr>
<th>Mesh</th>
<th>( \Delta_{wall}[\text{mm}] )</th>
<th>( N_r )</th>
<th>( N_\theta )</th>
<th>( Nu )</th>
<th>( T^*_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>0.4</td>
<td>61</td>
<td>401</td>
<td>22.10</td>
<td>0.6416</td>
</tr>
<tr>
<td>Medium</td>
<td>0.2</td>
<td>101</td>
<td>801</td>
<td>23.03</td>
<td>0.6333</td>
</tr>
<tr>
<td>Fine</td>
<td>0.05</td>
<td>121</td>
<td>1201</td>
<td>23.11</td>
<td>0.6346</td>
</tr>
</tbody>
</table>

**Figure 3: Mesh Resolution Effect on the Hydra Solutions for \( Ra = 10^8 \) (Rotor Revolution Time=2\pi)**

**Table 3: Mesh Information of the Test Cases Studied. \( \Delta_{wall} \): Near Wall Grid Height; \( N_r \): Radial Grid Node Number; \( N_\theta \): Tangential Grid Node Number; \( N_{tot} \): Total Grid Node Number; \( N_{el} \): Number of Elements; \( P \), Polynomial Order.**

<table>
<thead>
<tr>
<th>Hydra</th>
<th>( Ra )</th>
<th>( \Delta_{wall}[\text{mm}] )</th>
<th>( N_r )</th>
<th>( N_\theta )</th>
<th>( N_{tot} )</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^7</td>
<td>0.2</td>
<td>101</td>
<td>801</td>
<td>~0.08M</td>
<td>DNS</td>
<td></td>
</tr>
<tr>
<td>10^8</td>
<td>0.1</td>
<td>121</td>
<td>1201</td>
<td>~0.15M</td>
<td>DNS</td>
<td></td>
</tr>
<tr>
<td>3.3 \times 10^8</td>
<td>0.05</td>
<td>141</td>
<td>1401</td>
<td>~0.20M</td>
<td>DNS</td>
<td></td>
</tr>
<tr>
<td>10^9</td>
<td>0.03</td>
<td>121</td>
<td>1001</td>
<td>~0.12M</td>
<td>LES</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Semtex</th>
<th>( Ra )</th>
<th>( \Delta_{wall}[\text{mm}] )</th>
<th>( N_{el} )</th>
<th>( P )</th>
<th>( N_\theta )</th>
<th>( N_{tot} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^7</td>
<td>0.036</td>
<td>16</td>
<td>10</td>
<td>256</td>
<td>~0.04M</td>
<td></td>
</tr>
<tr>
<td>10^8</td>
<td>0.023</td>
<td>40</td>
<td>5</td>
<td>1024</td>
<td>~0.20M</td>
<td></td>
</tr>
<tr>
<td>3.3 \times 10^8</td>
<td>0.023</td>
<td>40</td>
<td>5</td>
<td>1024</td>
<td>~0.20M</td>
<td></td>
</tr>
<tr>
<td>10^9</td>
<td>0.023</td>
<td>40</td>
<td>5</td>
<td>1600</td>
<td>~0.32M</td>
<td></td>
</tr>
</tbody>
</table>
**Instantaneous Flow Field**

Examples of the instantaneous temperature fields obtained with Semtex, and normalised as \( T^* = (T - T_a)/(T_b - T_a) \), are given in Figure 4. Five pairs of cold/hot plumes or arms are observed for all the three conditions illustrated, and the flow state becomes more chaotic as \( Ra \) increases. At low \( Ra \) value strong arms penetrate through the core, driving the heat exchange. At high \( Ra \) value the arms are less visible, with shed vortices transporting heat between the shaft and shroud. Similar flow features and trends were observed in the Hydra solutions.

![Figure 4 Normalised Instantaneous Temperature Field](image)

**Shroud Heat Transfer**

The shroud Nusselt numbers predicted with both solvers are plotted against \( Ra \) in Figure 5, for Bohn et al.’s rig conditions. The graph also includes the correlation obtained from the rig by Bohn and co-workers (Bohn et al., 1995; Bohn and Gier, 1998), and a correlation acquired for heat convection between differentially heated horizontal plates under gravity by Hollands et al. (1975). The difference in \( Nu \) between the two codes is given, showing reasonable agreement, within a 10% threshold. The best agreement is observed at \( Ra = 10^8 \). Among the three correlations, the closest agreement with the predicted results is given by Bohn and Gier’s corrected experimental correlation. It should be noted that some differences are seen in comparison with the 3D tests reported by Pitz et al. (2019), indicating further study is needed to investigate the effects of the disc boundary layers.

![Figure 5 Shroud Nusselt Number versus Rayleigh Number. Comparison between Hydra and Semtex.](image)

**Shroud and Shaft Boundary Layers**

The shroud and shaft boundary layers are considered in this section, again presenting results at Bohn et al.’s rig conditions. Mean values presented here have been averaged both in time and in the circumferential direction.

**Thermal boundary layers**

Mean temperature profiles are given in Figure 6. Both codes predict a uniform mean core temperature, and the boundary layer thins as \( Ra \) increases, agreeing with Pitz et al.’s (2019) observation. In Semtex solutions \( T_{core}^* \) is nearly invariant over the range of \( Ra \) considered. This differs from Pitz et al.’s (2019) 3D simulations, in which \( T_{core}^* \) increases slightly with \( Ra \). For Hydra results a slight decrease of \( T_{core}^* \) is observed as \( Ra \) is increased.

Figure 7 shows the root mean square profiles of the temperature fluctuations. When \( Ra \) increases the \( T_{rms}^* \) peaks move toward the cylindrical surfaces, and the corresponding amplitudes reduce. This is observed in both solvers’ results. Some slight differences may be seen in results from the two solvers, although trends are generally similar.

![Figure 6 Mean Temperature Profiles for Bohn et al.’s Rig Conditions, Averaged in Time and in the Tangential Direction](image)

**Kinematic boundary layers**

The simulations are run in a relative frame of reference rotating with the rig and the mean flow is close to solid body rotation. Hence, the velocity fluctuation profiles are plotted
here to investigate the kinematic boundary layer. The root mean squares of $v_\theta$ are shown in Figure 8, with different normalisations ($\Omega a$ and $\Omega a \sqrt{\beta \Delta T}$). Two peaks are observed near the cylindrical surfaces. In subplot (a) stronger peak values are given by low $Ra$ condition. Taking $\Omega a \sqrt{\beta \Delta T}$ to scale the velocity fluctuations, as shown in subplot (b), reasonably good agreement of the curves is achieved, particularly for the peak amplitudes. This suggests that $\Omega a \sqrt{\beta \Delta T}$ is an appropriate velocity scale, as might be expected from the form of the driving centrifugal buoyancy force. However, this scale is not that successful in correlating the radial velocity fluctuations, as shown in Figure 9.

**Figure 8 Root Mean Square of Tangential Velocity Fluctuations for Bohn et al.’s Rig Conditions, Obtained with Hydra, Averaged in Time and in the Tangential Direction. (a) Normalised with $\Omega a$; (b) Normalised with $\Omega a \sqrt{\beta \Delta T}$**

**Figure 9 Root Mean Square of Radial Velocity Fluctuations for Bohn et al.’s Rig Conditions, Obtained with Hydra, Averaged in Time and in the Tangential Direction. (a) Normalised with $\Omega a$; (b) Normalised with $\Omega a \sqrt{\beta \Delta T}$**

**Effect of $\beta \Delta T$ for a Given Rayleigh Number**

From the definition in Eq. (1), it is clear that a given $Ra$ can be achieved by different combinations of $\beta \Delta T$ and $Re_\phi$. Figure 10 shows how the buoyancy parameter $\beta \Delta T$ affects the shroud $Nu$ for $Ra$ fixed at $3.3 \times 10^8$. No distinguishable difference is observed with Semtex solutions. Hydra results show the shroud $Nu$ decreases with the increase of $\beta \Delta T$ at high $\beta \Delta T$ values. This is similar to the behaviour expected in gravitational convection where the Boussinesq approximation is typically assumed to hold for $\beta \Delta T \leq 0.2$.

**Figure 10 Shroud $Nu$ for $Ra = 3.3 \times 10^8$, Obtained from Both Hydra and Semtex**

The profiles of the thermal boundary layers are shown in Figure 11. Some effects of $\beta \Delta T$ can be observed through the mean core temperature, and differences in the temperature fluctuations are just visible. Figure 12 plots the profiles of kinematic boundary layers. The profiles are normalised with $\Omega a \sqrt{\beta \Delta T}$. Here use of this parameter helps to collapse the plots of both the tangential and radial velocities.

**Figure 11 Mean Temperature and RMS of Temperature Fluctuations Profiles, Obtained with Hydra, $Ra = 3.3 \times 10^8$**

**Figure 12 RMS of Tangential and Radial Velocity Fluctuations, Obtained with Hydra, $Ra = 3.3 \times 10^8$.**

**Effect of Property Variation**

The fluid viscosity and thermal conductivity are known to vary with temperature, while these properties are treated as constants in the incompressible solver. A simulation was conducted in Hydra fixing the dynamic viscosity throughout the computational domain with $Ra = 3.3 \times 10^8$ and
$\beta \Delta T = 0.4$. A comparison with the varying viscosity test case at the same operating condition is shown in Figure 10 for shroud $Nu$. As the dynamic viscosity is fixed a smaller $Nu$ is obtained. The effect is very limited compared with the influence of $\beta \Delta T$ or $Ra$. For the profiles of thermal and kinematic boundary layers, the effect from fixing viscosity is considered negligible.

![Figure 13 Mean Temperature and RMS of Temperature Fluctuations Profiles, $Ra = 3.3 \times 10^8$, with Viscosity and Fixed Viscosity, Obtained from Hydra](image)

![Figure 14 RMS of Tangential and Radial Velocity Fluctuations, Obtained from Hydra. $Ra = 3.3 \times 10^8$, with Varying Viscosity and Fixed Viscosity](image)

**Centrifugal and Coriolis Forces**

The importance of Coriolis force in centrifugal buoyant flows in developing cyclonic and anti-cyclonic circulations for heat transport between the shaft and shroud has been emphasised by Owen and Long (2015). This force differentiates the present problem from natural heat convection between horizontal plates under gravity. The radial Coriolis force is induced by the relative circumferential flow motions and can act as a restoring force to weaken the heat transfer in the radial direction due to the centrifugal buoyancy force. Equation (6) gives the expressions of non-dimensional averaged centrifugal and radial Coriolis force terms from the compressible momentum equation in Hydra.

$$f_{cen} = \frac{(\rho - \rho_{core})\Omega^2 \tau}{(\rho_{core})\Omega^2 a}$$

$$f_{Cor} = 2(\rho \nu \psi)/(\rho_{core})\Omega^2 a$$  (6)

Figure 15 shows the centrifugal and radial Coriolis force for Bohn et al.’s rig conditions, obtained with Hydra. In the core the centrifugal forces are zero, corresponding to the uniform mean core temperatures. Close to the cylindrical surfaces large centrifugal forces are observed. These reduce with the increase of $Ra$, associated with the decrease of $\beta \Delta T$. The radial Coriolis forces show high values in the core and develop gradually to zero towards the cylindrical surfaces. Therefore, the flow is dominated by centrifugal buoyancy force near the shroud and shaft where the principal resistance to heat transfer occurs. This suggests Coriolis forces have little effect on overall heat transfer, contrasting with experimental and some numerical results that show lower average heat transfer than expected from gravitational convection correlations. This behaviour is being further investigated with full 3D simulations.

![Figure 15 Centrifugal and Radial Coriolis Forces for Rig Conditions, Obtained with Hydra.](image)

**CONCLUDING REMARKS**

Numerical studies have been conducted for buoyancy-driven flows in a closed rapidly rotating cavity using pseudo 2D models. Solvers for compressible and incompressible Navier-Stokes equations are considered. The incompressible solver employs the Boussinesq approximation in the centrifugal buoyancy term to consider the effect of density variation.

The study of mesh resolution effect with Hydra’s solutions shows how coarse meshes can weaken the “plumes”, thus under-predicting heat transfer.

Reasonably good agreement, within 10%, is achieved for shroud Nusselt number between the two solvers. Reasonably good agreement is also obtained, between the two solvers, for the thermal boundary layers. An invariant uniform mean core temperature is predicted by Semtex, differing from the 3D simulations by Pitz et al. (2019). Hydra shows uniform mean core temperatures but with temperature decreasing with the increase of $Ra$.

Regarding the kinematic boundary layers, the tangential velocity scales approximately with the parameter $\Omega a \sqrt{\beta \Delta T}$. But this does not apply to the radial velocity for the rig conditions considered.

Increasing $\beta \Delta T$ while at fixed $Ra$. Semtex gave negligible change in shroud $Nu$, whereas Hydra shows noticeable shroud $Nu$ reduction. This indicates the validity of the Boussinesq approximation weakens as $\beta \Delta T$ increases. Using a constant dynamic viscosity reduced slightly the shroud $Nu$. Note, however, that the variation of shroud $Nu$ for the conditions investigated ($Ra = 3.3 \times 10^8$,
\( \beta \Delta T \leq 0.4 \) is rather small compared with that produced by changing \( Ra \).

For a fixed \( Ra \) in Hydra, slight variation of mean core temperature was observed as \( \beta \Delta T \) changes. Both the tangential and radial velocities scaled approximately with the parameter \( \Omega \alpha / \sqrt{\beta \Delta T} \). Hydra solutions also show negligible differences on the thermal and kinematic boundary layers between varying and fixed viscosity.

The analysis of centrifugal and radial Coriolis forces from Hydra solutions suggests that the flow is dominated by the centrifugal force near the cylindrical surfaces, where the main resistance to heat transfer occurs.

Further study will consider full 3D simulations.

**NOMENCLATURE**

**Roman symbols**

- \( a, b \): Radii of the shaft and shroud
- \( Ec \): Eckert number \( Ec = \Omega^2 b^2 / (C_p \Delta T) \)
- \( f_{cen} \): Centrifugal force
- \( f_{Cor} \): Coriolis force
- \( Nu \): Shroud Nusselt number, ratio of convective heat flux to natural heat conduction
- \( Pr \): Prandtl number
- \( r^* \): Normalised radius \( r^* = (r - a) / (b - a) \)
- \( Ra \): Rayleigh number \( Ra = 2Pr \beta \Delta T Re \phi \frac{b - a}{b + a} \)
- \( Re \phi \): Rotational Reynolds number \( Re \phi = \frac{\rho \Omega (b + a)(b - a)}{2 \mu} \)
- \( t \): Non-dimensional time \( t = time / \Omega \)
- \( T^* \): Normalised temperature \( T^* = (T - T_a) / \Delta T \)
- \( T \): Mean temperature, averaged in time and the tangential direction
- \( T_{rms} \): Root mean square of temperature fluctuations
- \( \nu_{rms} \): Root mean square of radial velocity
- \( \nu_{\phi, rms} \): Root mean square of tangential velocity

**Greek symbols**

- \( \beta \): Thermal expansion coefficient \( \beta = 1 / (T_a + T_b) \)
- \( \beta \Delta T \): Buoyancy parameter
- \( \Delta T \): Temperature difference between shroud and shaft \( \Delta T = T_b - T_a \)
- \( \mu \): Dynamic viscosity
- \( \Omega \): Angular speed
- \( \rho \): Fluid density

**Subscripts**

- core: Value at the core

**REFERENCES**


